

- (1) A fair die is rolled and a fair coin is flipped.
  - (a) Write down a sample space for this experiment.
  - (b) Write down the event (as a set) that the coin lands heads.
  - (c) Name the events which have the smallest and largest probabilities.
  - (d) Write down the following event and compute its probability:  
(the coin is heads and the die roll is odd, but not three) or (the die roll is three).
  - (e) Careless writing makes the following description ambiguous:  
The die is 3, or even, and the coin is heads, or the coin is tails, or the die is 4.  
Write all possible interpretations (i.e. distinct answers stemming from adding the needed brackets) and their probabilities.
- (2) If the sample space  $\Omega$  has  $n$  elements, what is the total number of possible events?
- (3) A fair coin is tossed and a subset of  $\{0, 1, 2\}$  is chosen at random. All outcomes are equally likely.
  - (a) Write down a suitable sample space.
  - (b) What is the probability that the chosen subset contains the element 0?
- (4) A number is to be chosen at random from the range  $0 \leq x \leq 1$ .
  - (a) Choose an appropriate sample space.
  - (b) We write  $x$  as a decimal (without recurring 9). Write down the event that the first or second digit (after the point) is a three.
- (5) A number is to be chosen at random from the range  $0 \leq x \leq 1$ . It is given that
$$\mathbb{P}(\{x : a \leq x \leq b\}) = b - a$$
for every pair  $0 \leq a < b \leq 1$ . Use Problem 13 and the axioms to show:
  - (a)  $\mathbb{P}(\{x : 0 < x < \frac{1}{2}\}) = \frac{1}{2}$ .
  - (b)  $\mathbb{P}(\{x : x = \frac{1}{3}\}) = 0$ .
- (6) Problem 6 from Chapter 1.
- (7) Problem 8 from Chapter 1.
- (8) (a) Starting from the axioms, show that for any events  $A_1, A_2, \dots, A_n$  we have
$$\mathbb{P}(A_1 \cup A_2 \cup \dots \cup A_n) \leq \mathbb{P}(A_1) + \mathbb{P}(A_2) + \dots + \mathbb{P}(A_n).$$
We will refer to this inequality as the *union bound*.
  - (b) Use this result to resolve Problem 11 from Chapter 1.

Continued...

- <sup>2</sup> (9) Study problem 12 from Chapter 1.
- (10) Suppose we choose an integer  $1 \leq n \leq 10$  at random, where each answer is equally likely. Compute the probability that  $n$  is a prime number using inclusion/exclusion and the fact that
- $n$  is *not* prime  $\Leftrightarrow n = 1$ , or  $n$  is a multiple of 2 (other than 2 itself), or  
 $n$  is a multiple of 3 (other than 3), or  
 $n$  is a multiple of 5 (other than 5), or ...