170A Killip Midterm October 26th

First Name: ____________________ ID#____________________

Last Name: ____________________

Rules.

- There are **FOUR** problems; ten points per problem.
- There are extra pages after every problem. You may also use the backs of pages.
- No calculators, computers, notes, books, crib-sheets,...
- Out of consideration for your class-mates, no chewing, humming, pen-twirling, snoring,... Try to sit still.
- Turn off your cell-phone, pager,...

1	2	3	4	$\sum$

(1) (a) Define the term *event* as used in probability theory.

(b) Give an example of an experiment that results in a random variable with the Binomial($\frac{1}{2}, 7$) law.

We have four trash bins: one blue, one green, and two indistinguishable black bins.

(c) How many distinct ways can these be lined up in the street?

(d) If all such arrangements are equally likely, what is the probability that the two black bins are next to each other?

$\boxed{\text{extra paper}}$

(2) (a) State the axioms that a *probability law* $\mathbb{P}$ on a sample space Ω must obey.

(b) Define what it means for events A and B to be *independent*.

(c) Use the information above to verify the following:

If A and B are independent, then A and B^c are independent.

Justify each step.

$\boxed{\text{extra paper}}$

(3) A zoo has two female ostriches, one named α, the other named β.
Each breeding season, the ostriches produce a random number of eggs:
For α, the number of eggs has a Geometric distribution and she lays 2 eggs on average.
For β, the number of eggs has a Poisson distribution; again, she lays 2 eggs on average.

(a) What is the average total number of eggs laid by both ostriches?
(b) What is the parameter p associated to α's Geometric distribution?
(c) What is the probability that β lays exactly two eggs?
(d) I observe that one of the nests (chosen at random) has exactly two eggs.
What is the probability this nest belongs to α?

$\boxed{\text{extra paper}}$

(4) The table below gives the values of the joint PMF for the random variables X and Y. All combinations of values not listed have probability zero.

	$X = 0$	$X = 1$
$Y = 0$	$\frac{1}{12}$	$\frac{1}{4}$
$Y = 1$	$\frac{1}{6}$	$\frac{1}{6}$
$Y = 2$	$\frac{1}{3}$	0

Determine each of the following:
(a) $\mathbb{E}(X)$.
(b) The covariance of X and Y.
(c) The PMF of the random variable Z defined as the minimum of X and Y.

$\boxed{\text{extra paper}}$