As usual, M is a fixed countable transitive model of ZFC and $\mathbb{P}$ is a fixed poset such that $\mathbb{P} \in M$.

Problem 1. Fix a $\mathbb{P}$-generic filter G. Evaluate each:

1. $\sigma_0[G]$, where $\sigma_0 = \varnothing$.
2. $\sigma_1[G]$, where $\sigma_1 = \{\langle \sigma_0, \mathbb{1} \rangle\}$.
3. $\sigma_2[G]$, where $\sigma_2 = \{\langle \sigma_0, \mathbb{1} \rangle, \langle \sigma_1, \mathbb{1} \rangle\}$.
4. Show that for all $n \in \omega$ there is a name ρ such that $\rho[G] = n$, regardless of what G is.
5. Show that there is a name τ such that $\tau[G] = \omega$, regardless of what G is.

Problem 2. Suppose that $A \subseteq \mathbb{P}$ is an antichain. Suppose that for each $p \in \mathbb{P}$, τ_p is a $\mathbb{P}$-name. Let G be a filter. What can we say about the cardinality of $\pi[G]$, where $\pi = \{\langle \tau_p, p \rangle : p \in A\}$?

Problem 3. Let $\mathbb{P} = \mathrm{Fn}(\omega, \omega)$. For each $n \in \omega$, let $D_n = \{p \in \mathbb{P} : n \in \mathrm{dom}(p)\}$. Let $\mathcal{D} = \{D_n : n \in \omega\}$. Then the usual arguments from the Martin's Axiom portion of the class tell us that given a $\mathcal{D}$-generic filter G, $g = \bigcup G$ yields a function $g : \omega \to \omega$. In the following, I am asking for examples of a fixed concrete name that will work regardless of which $\mathcal{D}$-generic filter G actually is.

1. Give an example of a $\mathbb{P}$-name τ such that $\tau[G] = \{\varnothing\}$ if $g(0) = 0$ and $\tau[G] = \varnothing$ otherwise.
2. Give an example of a $\mathbb{P}$-name τ such that $\tau[G] = \{\varnothing\}$ if 0 is in the range of g and $\tau[G] = \varnothing$ otherwise.
3. Give an example of a $\mathbb{P}$-name τ such that $\tau[G] = \omega$ exactly when g is surjective.
4. Give a $\mathbb{P}$-name τ such that $\tau[G] = \{g(0)\}$.
5. Give a $\mathbb{P}$-name τ such that $\tau[G]$ is equal to the set $g^{-1}[0]$.
6. Give a $\mathbb{P}$-name τ such that $\tau[G] = g(0)$.
7. Show that there is no $\mathbb{P}$-name τ such that $\tau[G] = \{\varnothing\}$ if g is surjective and $\tau[G] = \varnothing$ otherwise.

Problem 4. Suppose that $\mathbb{P}$ is not nonatomic. Then there is a $\mathbb{P}$-generic filter G with G in M.