

Problem 1. Let $\mathbb{P} \in M$ be a separative partial order, and Γ the canonical name for the generic filter. Show that $q \Vdash \check{p} \in \Gamma$ if and only if $q \leq p$.

Problem 2. An automorphism $i : \mathbb{P} \rightarrow \mathbb{P}$ is a bijection such that $p \leq q$ if and only if $i(p) \leq i(q)$. A poset is *almost homogeneous* if for any $p, q \in \mathbb{P}$ there is an automorphism $i : \mathbb{P} \rightarrow \mathbb{P}$ such that $i(p)$ and q are compatible. Do the following:

1. Show that if I is an infinite set and J is any set then $\text{Fn}(I, J)$ is almost homogeneous.
2. Given an automorphism $i : \mathbb{P} \rightarrow \mathbb{P}$ with $i \in M$ and a \mathbb{P} -name τ we define recursively the \mathbb{P} -name $i(\tau)$ by letting $i(\tau)$ be all the pairs $\langle i(\sigma), i(p) \rangle$ where $\langle \sigma, p \rangle \in \tau$. Show that for any $x \in M$, $i(\check{x}) = \check{x}$.
3. Let $\tau_1, \dots, \tau_n \in M^{\mathbb{P}}$. Show that $p \Vdash \phi(\tau_1, \dots, \tau_n)$ if and only if $i(p) \Vdash \phi(i(\tau_1), \dots, i(\tau_n))$.
4. Suppose that \mathbb{P} is almost homogeneous, and let G be \mathbb{P} -generic. Let $x_1, \dots, x_n \in M$. Show that if $M[G] \models \phi(x_1, \dots, x_n)$ then in fact $\mathbb{1} \models \phi(\check{x}_1, \dots, \check{x}_n)$.
5. Conclude that if \mathbb{P} is almost homogeneous, then for any \mathbb{P} -generic filters G and H , $M[G]$ and $M[H]$ are elementarily equivalent (with respect to first order logic).

Problem 3. Let \mathbb{P} be a poset, and let \mathbb{Q} be a dense set of \mathbb{P} , viewed as a subposet. Do the following.

1. If G is \mathbb{P} -generic, then if we set $H = G \cap \mathbb{Q}$, we have that H is \mathbb{Q} -generic. Further, $G = \{p \in \mathbb{P} : (\exists q \in H) q \leq p\}$.
2. If H is \mathbb{Q} -generic, then if we set $G = \{p \in \mathbb{P} : (\exists q \in H) q \leq p\}$ then G is \mathbb{P} -generic. Further, $H = G \cap \mathbb{Q}$.
3. If G and H are taken as in either of the two above, then $M[G] = M[H]$.
4. Any \mathbb{Q} -name is a \mathbb{P} -name, and if $\tau_1, \dots, \tau_n \in M^{\mathbb{Q}}$ then $\Vdash_{\mathbb{P}} \phi(\tau_1, \dots, \tau_n)$ if and only if $\Vdash_{\mathbb{Q}} \phi(\tau_1, \dots, \tau_n)$.
5. For any $\tau \in M^{\mathbb{P}}$ there is a $\sigma \in M^{\mathbb{P}}$ such that $\Vdash_{\mathbb{P}} \tau = \sigma$.

Problem 4. Two forcing notions \mathbb{P} and \mathbb{Q} are forcing equivalent if there are $\tau \in \mathbb{P}$ and $\sigma \in \mathbb{Q}$ such that

- (a) $\Vdash_{\mathbb{P}} \tau$ is a \mathbb{Q} -generic filter.
- (b) $\Vdash_{\mathbb{Q}} \sigma$ is a \mathbb{P} -generic filter.
- (c) If G is \mathbb{P} -generic then $G = \sigma[\tau[G]]$.
- (d) If H is \mathbb{Q} -generic then $H = \tau[\sigma[H]]$.

Do the following:

1. Show that if \mathbb{Q} is a dense subset of \mathbb{P} , then the two are forcing equivalent.
2. Show that forcing equivalence is an equivalence relation.