Problem 1. As usual, κ, λ and μ denote cardinals.

1. Show that $(\kappa^\lambda)^\mu = \kappa^{(\lambda\cdot\mu)}$.
2. Show that $\kappa^\kappa = 2^\kappa$.

Problem 2. Let A be a given infinite set. Show that the set of all bijections from A into A has cardinality 2^κ.

Problem 3. Let κ an infinite cardinal, let λ be another cardinal with $\lambda \leq \kappa$. We write $[\kappa]^\lambda$ to denote the collection of all subsets of κ with cardinality λ. Show that $[\kappa]^\lambda$ has cardinality κ^λ.

Problem 4. The limit of cardinals is a cardinal.

Problem 5. Suppose that $\langle \alpha_i : i < \lambda \rangle$ is an increasing sequence of ordinals unbounded in κ. Then $\mathrm{cf}(\lambda) = \mathrm{cf}(\kappa)$.

Problem 6 (*). For a given set x, let $\mathcal{P}_{\mathrm{wo}}(x)$ denote the collection of all the subsets of x that can be well-ordered. Under the Axiom of Choice this is the same as $\mathcal{P}(x)$, but without the Axiom of Choice the two may well be different. Nonetheless, show without appealing to the Axiom of Choice that there is no injection from $\mathcal{P}_{\mathrm{wo}}(x)$ into x.

Problem 7 (*). Let λ be an infinite cardinal. Say that a family F of functions $f : \lambda \to \lambda$ covers λ if for any distinct $\alpha, \beta < \lambda$ there is an $f \in F$ such that $f(\alpha) = \beta$ or $f(\beta) = \alpha$. Show that for every cardinal κ there is a family of size κ covering κ^+ but no family of smaller size does so.

Problem 8 (*). Say that a set $R \subseteq \omega_1 \times \omega$ is a *large rectangle* if it has the form $A \times B$ for some uncountable A and infinite B. Assume CH. Then there exists a set $S \subseteq \omega_1 \times \omega$ which neither contains a large rectangle nor is disjoint from one.

Problem 9 (*). Let κ be an infinite cardinal and $\lhd$ a well-ordering on κ. Show that there is an $X \subseteq \kappa$ such that $|X| = \kappa$ and $\lhd$ and $<$ agree on X.