Problem 1. Say a function $f : \omega \to \omega$ which never takes the value zero is *big* if $\sum_{i<\omega} \frac{1}{f(i)} < 1$. Assume $MA(\kappa)$, and let $\{f_\alpha : \alpha < \kappa\}$ be a family of big functions. Show that there is a big function g such that $g <^* f_\alpha$ for all $\alpha < \kappa$.

Problem 2. Say that a set $R \subseteq \omega_1 \times \omega$ is a *large rectangle* if it has the form $A \times B$ for some uncountable A and infinite B. Assume $\mathrm{MA}(\omega_1)$. Then every $S \subseteq \omega_1 \times \omega$ contains a large rectangle or is disjoint from one.

Problem 3. Given an almost disjoint family we say that an infinite set $Y \subseteq \omega$ *avoids* $\mathcal{A}$ if for any $A_0, \ldots A_n$ belonging to $\mathcal{A}$ we have that $Y \setminus (A_0 \cup \ldots \cup A_n)$ is infinite. We say an almost disjoint family $\mathcal{A}$ is *strongly MAD* if for every sequence $\langle Y_n : n \in \omega \rangle$ of sets avoiding $\mathcal{A}$ there is some $X \in \mathcal{A}$ such that $X \cap Y_n$ is infinite for each $n \in \omega$. Show that MA implies the existence of a strongly MAD family.

Problem 4. For each $n \in \omega$ say that an almost disjoint family $\mathcal{A}$ is n-strongly MAD if for every sequence $\langle Y_k : k < n \rangle$ of sets avoiding $\mathcal{A}$ there is some $X \in \mathcal{A}$ such that $X \cap Y_k$ is infinite for each $k < n$. Show MA implies that for each n there exists an n-strongly MAD family which is not $n + 1$-strongly MAD.

Problem 5 (Open Question)**.** Can one construct in ZFC a strongly MAD family? Or even a 2-strongly MAD family?