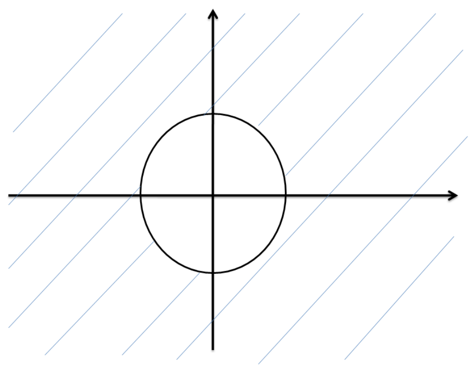
Math 32 A: Practice Midterm 2

1. Shade the region of the xy plane representing the domain of the following functions?

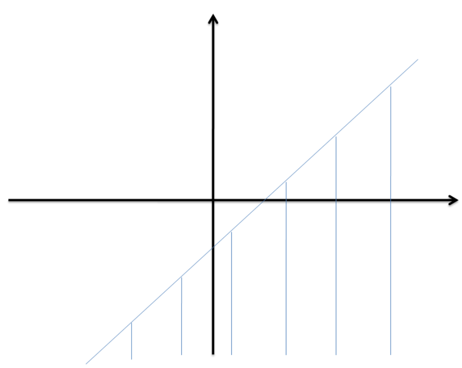
   a. $f(x,y) = \ln(x^2 + y^2 - 25)$

$$x^2 + y^2 \geq 25$$



   b. $f(x,y) = \sqrt{\ln(x-y)}$

$$x - y \geq 1$$



2. Let $f(x) = \begin{cases} 3x+2 & \text{if } x \leq 2 \\ 4x & \text{if } x > 2 \end{cases}$.

   Show that for all $\epsilon > 0$, there is a $\delta > 0$ such that $|f(x) - 8| < \epsilon$, if $|x - 2| < \delta$.

   (i) $x \leq 2 :\ |f(x) - 8| = |3x + 2 - 8| = 3|x - 2| < 3\delta$

   (ii) $x > 2 :\ |f(x) - 8| = |4x - 8| = 4|x - 2| < 4\delta$

   So $\delta = \frac{\epsilon}{4}$ will work in both cases.

3. The kinetic energy of a body with mass m and velocity v is

$$K(m, v) = \frac{1}{2}mv^2.$$

Show that

$$\frac{\partial K}{\partial m}\frac{\partial^2 K}{\partial v^2} = K(m, v).$$

$$\frac{\partial K}{\partial m} = \frac{1}{2}v^2, \quad \frac{\partial^2 K}{\partial v^2} = m$$

So $\frac{\partial K}{\partial m}\frac{\partial^2 K}{\partial v^2} = \frac{1}{2}v^2 m = K(m, v)$

4. Use implicit differentiation of find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ where

$$\sin(xyz) = x + 2y + 3z$$

$$\cos(xyz)(xy\frac{\partial z}{\partial x} + yz) = 1 + 3\frac{\partial z}{\partial x}$$

$$(xy\cos(xyz) - 3)\frac{\partial z}{\partial x} = 1 - zy\cos(xyz)$$

$$\frac{\partial z}{\partial x} = \frac{1 - zy\cos(xyz)}{(xy\cos(xyz) - 3)}$$

Similarly,

$$\frac{\partial z}{\partial y} = \frac{2 - zx\cos(xyz)}{(xy\cos(xyz) - 3)}$$

5. Is $f(x, y) = \sqrt{x + e^{4y}}$ differentiable at the point (3,0)? If so, find the linearization of f at (3,0).

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x + e^{4y}}}, \quad \frac{\partial f}{\partial y} = \frac{2e^{4y}}{\sqrt{x + e^{4y}}}$$

Both of these are continuous at (3,0) so the function is differentiable. The linearization is

$$L(x, y) = f(3, 0) + \frac{\partial f}{\partial x}(3, 0)(x - 3) + \frac{\partial f}{\partial y}(3, 0)y = 2 + \frac{1}{4}(x - 3) + y.$$