

Math 32 A: Practice Midterm 2 Solutions

1. What are the domain and range of the following functions?

a. $f(x, y) = 4x^2 + y^2 + 1$

Domain: All points (x, y) , Range: $\{z | z \geq 1\}$

b. $f(x, y) = \frac{1}{\ln(x+y^2)}$

Domain: $\{(x, y) | x > 0, y^2 \neq -\ln(x)\}$, Range: $\{z | z \neq 0\}$

2. Find an equation for the surface obtained by rotating the line $z = 4y$ about the z axis. What type of surface is this?

If we look in the zy plane at the line $z = 4y$ and consider the point $(0, y, 4y)$ we see that it follows a circular path as we rotate the curve about the z axis. This circle has radius $r = y = \frac{z}{4}$ so $x^2 + y^2 = r^2 = (\frac{z}{4})^2$ is the equation for the surface of rotation (i.e. it's a cone).

3. Find the limit of the following functions if they exist

a. $\lim_{(x,y) \rightarrow (1,2)} \frac{e^{xy}}{x^2 + 5y^3}$

Continuous functions in the denominator and numerator and non-zero denominator imply this is a continuous function. So the limit is equal to the function value.

$$\lim_{(x,y) \rightarrow (1,2)} \frac{e^{xy}}{x^2 + 5y^3} = \frac{e^2}{41}$$

b. $\lim_{(x,y) \rightarrow (1,2)} \frac{\sin(x)}{(x-1)^2 + 5(y-2)^3}$

Approaching the point $(1,2)$ along the y axis (i.e. $x = 1$) we see that $f(1, y) = \frac{\sin(1)}{5(y-2)^3}$. However, this will become unbounded as y approaches 2 so the limit cannot exist.

4. Show that the function $u(x, y, t) = e^{-\mu t} \sin(x) \cos(y)$ satisfies the heat equation

$$u_t = \frac{\mu}{2}(u_{xx} + u_{yy}).$$

$$u_t = -\mu e^{-\mu t} \sin(x) \cos(y), \quad u_{xx} = -e^{-\mu t} \sin(x) \cos(y), \quad u_{yy} = -e^{-\mu t} \sin(x) \cos(y)$$

so

$$\frac{\mu}{2}(u_{xx} + u_{yy}) = -\mu e^{-\mu t} \sin(x) \cos(y) = u_t.$$

5. a. Find an equation for the tangent plane to the surface given by $f(x, y) = \frac{\sin(x)}{x^2+3y^3}$ at the point $(x, y) = (1, 3)$.

$$\frac{\partial f}{\partial x} = \frac{\cos(x)(x^2 + 3y^3) - 2x\sin(x)}{(x^2 + 3y^3)^2}, \quad \frac{\partial f}{\partial y} = \frac{-9y^2\sin(x)}{(x^2 + 3y^3)^2}$$

Tangent plane:

$$z - \frac{\sin(1)}{82} = \frac{82\cos(x) - 2\sin(1)}{82^2}(x - 1) + \frac{-81\sin(1)}{82^2}(y - 3)$$

- b. Does the line parameterized as $\mathbf{r}(t) = (t, 3, 5t)$ lie in the tangent plane?

When $t = 1$ we see that $\mathbf{r}(1) = (1, 3, 5)$. Plugging into the equation for the tangent plane, we see that this point is not on the tangent plane and thus the line does not lie in the plane:

$$5 - \frac{\sin(1)}{82} \neq \frac{82\cos(x) - 2\sin(1)}{82^2}(1 - 1) + \frac{-81\sin(1)}{82^2}(3 - 3) = 0$$