

## Math 32 A: Practice Midterm 1

- 1. Determine whether the points  $P, Q, R$  lie on a straight line.
  - a.  $P = (2, 4, 2), Q = (3, 7, -2), R = (1, 3, 3)$
  - b.  $P = (0, -5, 5), Q = (1, -2, 4), R = (3, 4, 2)$
- 2. Suppose  $\mathbf{a}$  and  $\mathbf{b}$  are non-zero vectors.
  - a. Under what circumstances is  $\text{comp}_{\mathbf{a}}\mathbf{b} = \text{comp}_{\mathbf{b}}\mathbf{a}$ ?
  - a. Under what circumstances is  $\text{proj}_{\mathbf{a}}\mathbf{b} = \text{proj}_{\mathbf{b}}\mathbf{a}$ ?
- 3. Find the angle between the diagonal of a cube and one of its faces.
- 4. Find an equation for the plane defined by points equidistant from the points  $(1, 0, -2)$  and  $(3, 4, 0)$ .
- 5. Consider the points  $P$  such that the distance from  $P$  to the point  $A = (-1, 5, 3)$  is twice the distance to  $B = (6, 2, -2)$ . Show that the set of all such points is a sphere.
- 6. Consider a projectile shot with initial velocity  $\mathbf{v}_0$  above a slope with angle  $\theta$  from the horizontal. How long does it take the projectile to hit the slope? Write your answer in terms of gravitational constant  $g$ ,  $\theta$  and  $\mathbf{v}_0 = (v_{0x}, v_{0y})$ .

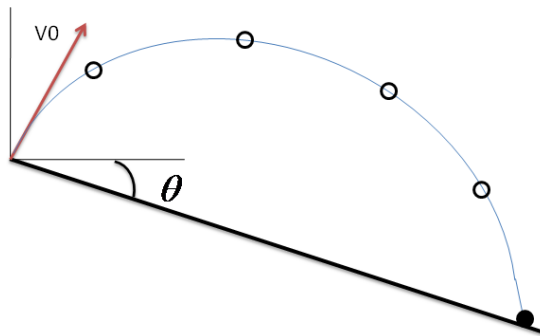


Figure 1: Problem 6 illustration

- 7. Consider an amusement park ride consisting of a large rotating wheel with rotating carriages that the passengers ride in. If the large wheel rotates with angular velocity  $\omega_1$  radians per second (i.e. as time varies from  $t = 0$  to  $t = 1$  an angle of  $\omega_1$  radians is swept out) and the carriage rotates with angular velocity  $\omega_2$ , write an equation for the position  $(\mathbf{r}(t))$  of a passenger's head assuming that the passenger boards the ride as shown in figure 2. Hint: this is very similar to the cycloid trajectory, see figure 2 below. Here, the position of the passenger's head is described similarly to the point on the rotating wheel for the cycloid.

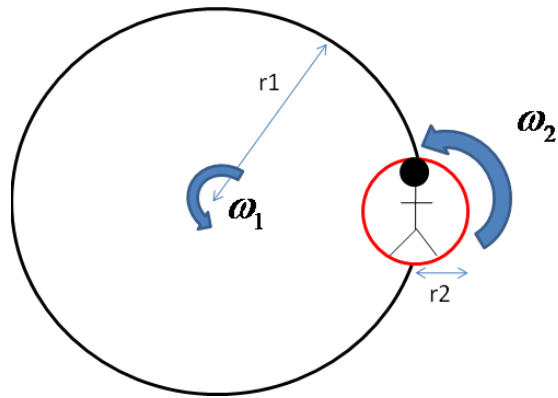


Figure 2: Problem 7 illustration. The passenger's head is initially at  $\mathbf{r}(0) = (r_1, 0) + (0, r_2)$