Math 32 A: Practice Final

1. Use vectors to decide whether the triangle with vertices $P(1,-3,-2)$, $Q(2,0,-4)$ and $R(6,-2,-5)$ is right-angled.

2. Reparameterize the curve $\mathbf{r}(t) = (t, \cos(2t), \sin(2t))$ with respect to arc length measured from the point $(0,1,0))$ in the direction of increasing t.

3. A projectile is shot from the origin with initial velocity $\mathbf{v}_0 = (u_0, v_0)$. Assuming that the projectile acceleration is only due to gravity (i.e. $\mathbf{a} = (0,-9.8)$), how large must v_0 be if the projectile is to reach a give height $h > 0$ at some time t?

4. Find and sketch the domain of $f(x,y) = \ln(x+y)\sqrt{y-x}$.

5. Find the limit if it exists otherwise show that it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y}{\sin^2(x)}$$

6. a. Write the linearization $L(x,y)$ of the function $z = f(x,y) = x^3y^4$ at the point $(1,1)$.

   b. At what values t does the curve $\mathbf{r}(t) = (t^2, 3t+1, t+2)$ intersect the plane defined by the linearization in part a?

7. Define $h(u,v,w) = z(x(u,v,w), y(u,v,w))$ with $z(x,y) = x^2 + xy^3$, $x(u,v,w) = uv^2 + w^3$, $y(u,v,w) = u + ve^w$. What are $\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}$ and $\frac{\partial h}{\partial z}$ when $u = 2$,$v = 1$ and $w = 0$?

8. Find the surface of revolution defined by rotating the curve $y^2 + \frac{z^2}{9} = 1$ about the z-axis.

9. Two level surfaces $f(x,y,z) = 0$ and $g(x,y,z) = 0$ meet in a curve at the point (a,b,c). How would you find the tangent to the curve at the point (a,b,c)?

10. Use Lagrange multipliers to find the extreme values of the function $f(x,y) = x^2+2y^2$ on the circle $x^2+y^2 = 1$.

11. Find the maximum and minimum values of $f(x,y) = xy - y + x$ on the set D which is the interior and boundary of the closed triangular region with vertices $(0,0)$,$(2,0)$ and $(0,3)$.