**Math 32 A: Practice Final**

1. Find the distance between the parallel planes $10x + 2y - 2z = 5$ and $5x + y - z = 1$.

2. a. Write a parameterization $\mathbf{r}(t)$ of the line through $(3, 4, 5)$ in the direction $(8, 4, 1)$.

   b. Reparameterize the line in terms of arclength measured from the point $(3, 4, 5)$ in the direction of increasing $t$.

3. Find the velocity $\mathbf{v}(t)$ and position $\mathbf{r}(t)$ of a particle given that $\mathbf{a}(t) = (2, 6t, 12t^2)$, $\mathbf{v}(0) = (1, 0, 0)$ and $\mathbf{r}(0) = (0, 1, -1)$.

4. If $z = f(x, y)$ and, where $x = r\cos(\theta)$ and $y = r\sin(\theta)$, show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} + \frac{1}{r}\frac{\partial z}{\partial r}$$

5. Find the maximum directional derivative of $f(x, y) = \frac{y^2}{x}$ at the point $(2, 4)$ and the direction in which it occurs.

6. Find the dimensions of a rectangular box with a volume of 32000 that has minimal surface area.

7. Use Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = x^2y^2z^2$ subject to the constraint $g(x, y, z) = x^2 + y^2 + z^2 = 1$

8. Suppose you are at the point $(a, b, c)$ on the level surface $f(x, y, z) = k$ and that $\nabla f(a, b, c) = (2, 3, -4)$.

   a. If $\mathbf{u}$ is tangent to the surface at $(a, b, c)$, what would $D_{\mathbf{u}}f(a, b, c)$ equal?

   a. If $\mathbf{u}$ is normal to the surface at $(a, b, c)$, what would $D_{\mathbf{u}}f(a, b, c)$ equal?

9. For which values of the constant $k$ does $x^2 + kxy + 3y^2$ have a relative minimum at $(0, 0)$?

10. Find the absolute maximum and minimum of $f(x, y) = 2x^3 + y^4$ over the set $D = \{(x, y)|x^2 + y^2 \leq 1\}$.