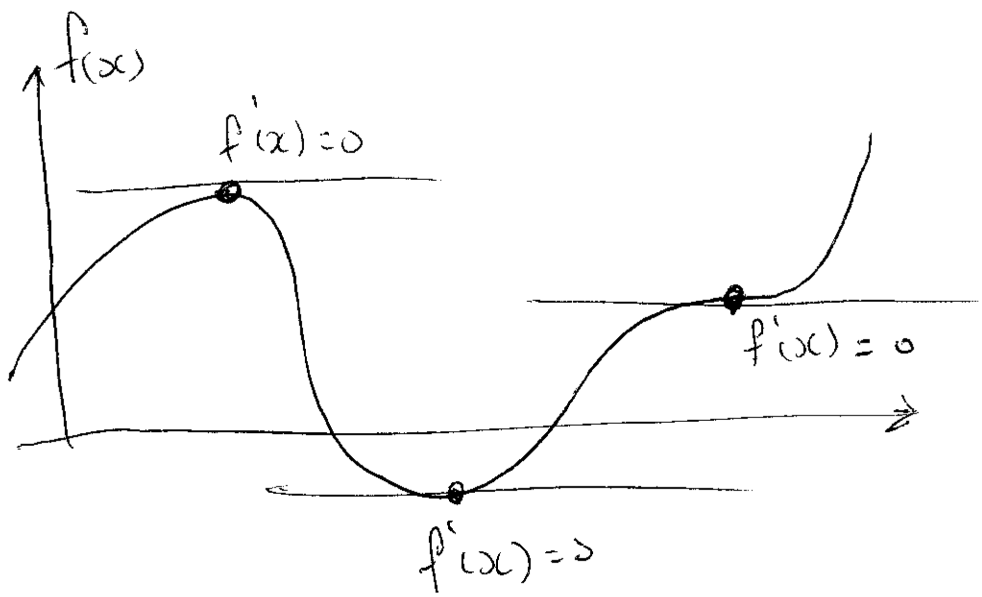
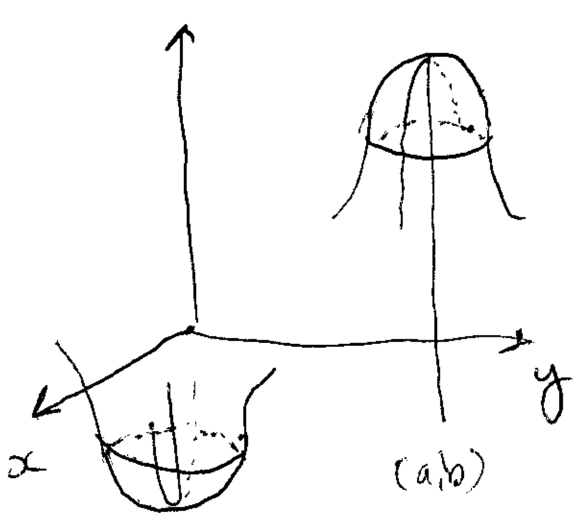
15.7 Maximum and minimum values



i, if $f'(x) = 0$ and $f''(x) < 0$, then f has a local min at x.

i if $f'(x) = 0$ and $f''(x) > 0$, then f has a local max at x.



Critical points:

$$\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = \underset{\sim}{0}$$

max and min criteria

$$D(x,y) = f_{xx}(x,y)\, f_{yy}(x,y) - f_{xy}^2(x,y)$$

Suppose the second partial derivatives of f are continuous on a disk w/ center (a,b).

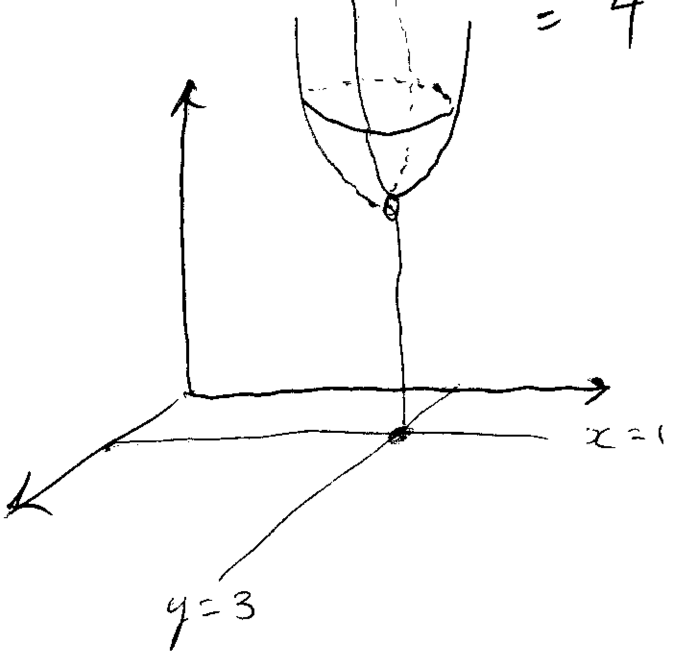
if $\nabla f(a,b) = \underset{\sim}{0}$ and

(i) if $D>0$ and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a local minimum.

(ii) if $D>0$ and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a local maximum.

(iii) if $D<0$, then $f(a,b)$ is not a local max or min.

E.g. $z = f(x,y) = x^2 + y^2 - 2x - 6y + 14$
$= 4 + (x-1)^2 + (y-3)^2$



$\nabla f = (2(x-1), 2(y-3))$

$= \underset{\sim}{0}$ if $x = 1$, $y = 3$

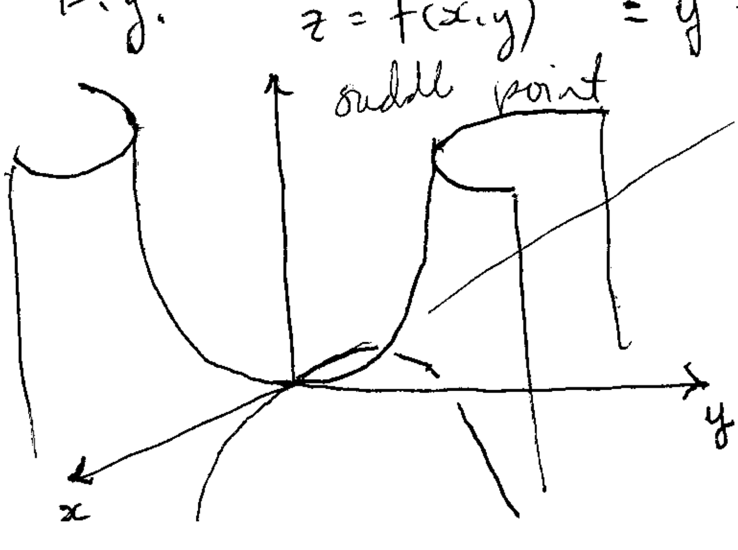
$f_{xx} = 2$ $\quad D = 4$

$f_{yy} = 2$

so $\nabla f(1,3) = \underset{\sim}{0}$ and $D = 4$ so

we have a local min

E.g. $z = f(x,y) = y^2 - x^2$



$\nabla f = (-2x, 2y)$

$\nabla f = (0,0)$ if $x = y = 0$

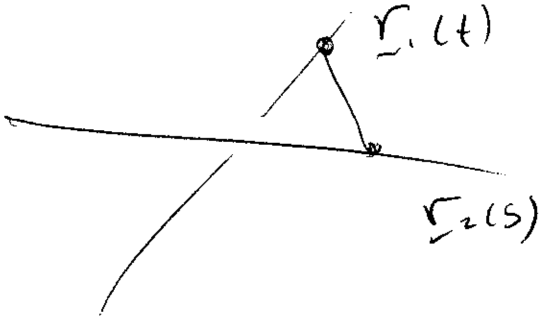
$f_{xx} = -2$ $\quad f_{xy} = 0$

$f_{yy} = 2$ $\quad D = -4$

4

$$\underset{\sim}{r}_1(t) = (0,1,3) + t(1,2,5) = (t, 1+2t, 3+5t)$$

$$\underset{\sim}{r}_2(s) = (0,0,1) + s(0,1,0) = (0, s, 1)$$



$$d(s,t) = \sqrt{(-t)^2 + (s-1-2t)^2 + (1-3-5t)^2}$$

$$d^2(s,t) = t^2 + (s-1-2t)^2 + (-2-5t)^2$$

$$\frac{\partial}{\partial t}(d^2) = 2t + 2(s-1-2t)(-2) + 2(-2-5t)(-5)$$

$$= 2t - 4(s-1-2t) + 10(2+5t)$$

$$\frac{\partial}{\partial s}(d^2) = 2(s-1-2t)(-2)$$

$$\therefore \quad s = 1+2t \quad \rightarrow \quad 2t + 20 + 50t = 0$$

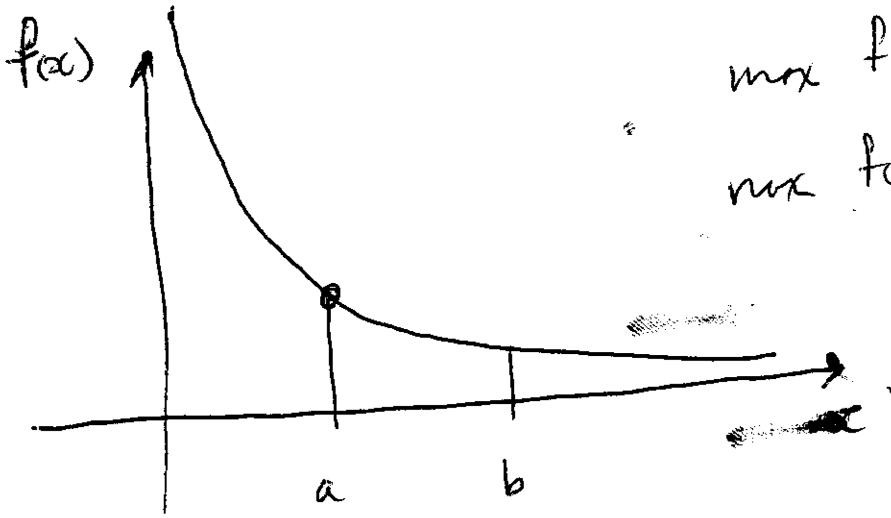
$$t = \frac{-20}{52} = \frac{-5}{13}$$

$$s = \frac{3}{13}$$

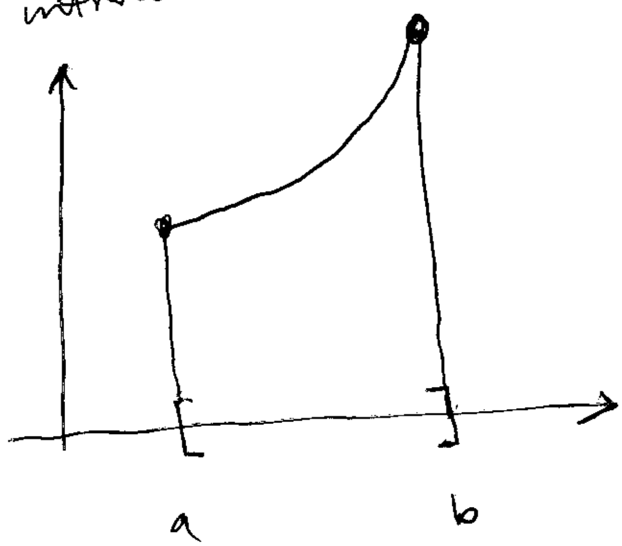
max and min over restricted domain

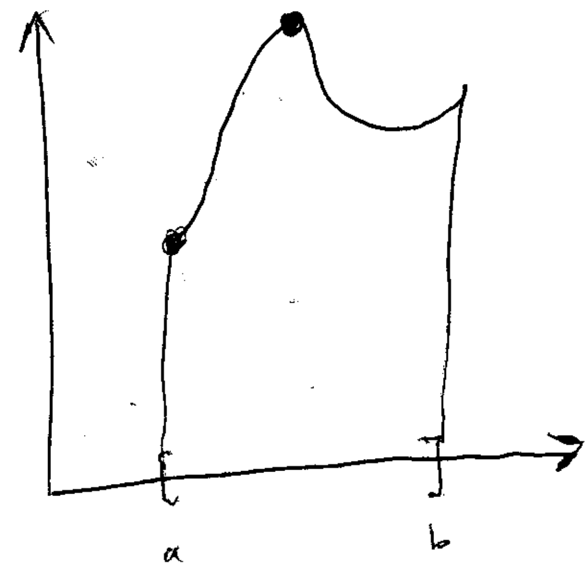
max $f(x)$ for x in $[a,b]$

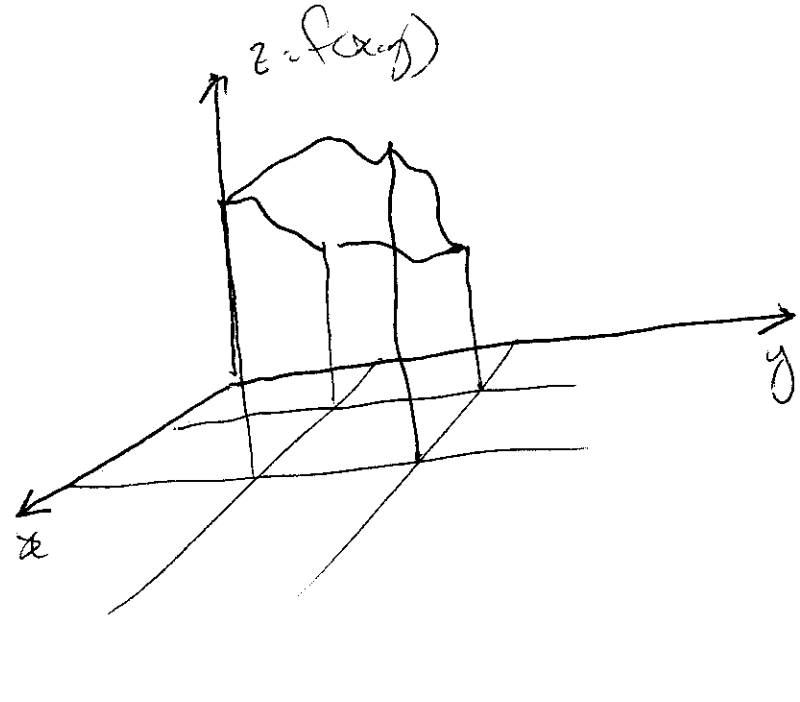
max $f(x)$ for x in $(0,b]$



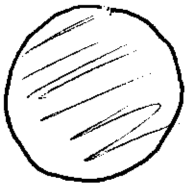
closed interval







Closed sets: contain their boundary



$D = \{(x,y) \mid x^2+y^2 \le 1\}$

Open sets: not closed

$D = \{(x,y) \mid x^2+y^2 < 1\}$

Extreme value theorem:

if f is continuous on a closed, bounded set D in $\mathbb{R}^2$, then f attains an

absolute max value $f(x_1, x_2)$ and

" min value $f(x_2, y_2)$ at

some points $(x_1, y_1) \in D$

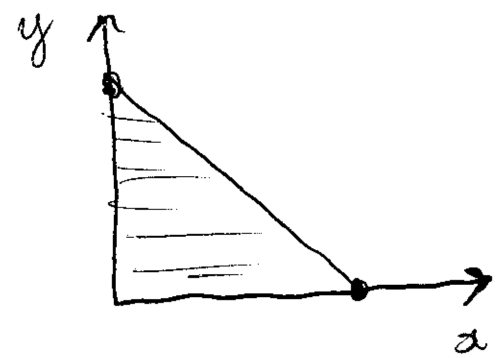
$(x_2, y_2) \in D$

E.g. Find the max and min of

$f(x,y) = x^2 + y^2$ on $D = \{ (x,y) \mid$

$y \geq 0$ $\quad y \leq 1 - x$

$x \geq 0$



$\nabla f = (2x, 2y)$

$\nabla f = \underline{0} \rightarrow x = y = 0$

global min

at $x = y = 0$

* if the global max is in the interior,

then $\nabla f = \underline{0}$ there

$\therefore$ global max must be on the boundary

To find max and min values of a continuous function f on a closed, bounded set D:

1. Find critical points of f in D ($\nabla f = \underset{\sim}{0}$)
2. Find extreme values of f on the boundary of D
3. largest from steps 1, 2 = absolute max

   smallest = min

E.g. Find the max and min of

$$f(x,y) = x^2 - 2xy + 2y \quad \text{on} \quad D = \{(x,y) \mid x \in (0,3),\ y \in (0,2)\}$$

$$\frac{\partial f}{\partial x} = 2x - 2y \qquad \frac{\partial f}{\partial y} = -2x + 2$$

$\rightarrow (x,y) = (1,1)$