HW3: Math 272A: Continuum Mechanics

Due Saturday, March 20

1. Show that the equations for linear elasticity

$$P_{ij} = 2\mu\epsilon_{ij} + \lambda\epsilon_{kk}\delta_{ij},\ \ \epsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$$

$$P_{ij,j} = f_i$$

can be rewritten as

$$u_{i,jj} + p_{,i} = \frac{f_i}{\mu}$$

$$-u_{k,k} + \left(\frac{\mu}{\lambda + \mu}\right) p = 0.$$

2. Show that by introducing variables $\mathbf{w}$ and q with the relation

$$u_i = w_i + q_{,i} \text{ and } p = q_{,kk}$$

that $\mathbf{w}$ and q satisfy

$$w_{i,jj} = \frac{f_i}{\mu}$$

$$-w_{k,k} + \left(\frac{\mu}{\lambda + \mu} - 1\right) q_{,kk} = 0$$

3. Show that the stress strain relationship is invertible for isotropic linear elasticity. Specifically, show that

$$\mathbf{P} = 2\mu\epsilon + \lambda \mathrm{tr}(\epsilon)\mathbf{I}$$

implies that

$$\epsilon = -\frac{\nu}{\gamma}\mathrm{tr}(\mathbf{P})\mathbf{I} + \frac{1+\nu}{\gamma}\mathbf{P}.$$

4. Assume we are given some tensor field over the undeformed configuration B_0:

$$\epsilon : B_0 \to \mathrm{V}^2.$$

Suppose we'd like to find a displacement field $\mathbf{u} : B_0 \to \mathrm{V}$ such that

$$\frac{1}{2}\left(u_{i,j}(\mathbf{X}) + u_{j,i}(\mathbf{X})\right) = \epsilon_{ij}(\mathbf{X}),\ \ \forall \mathbf{X} \in B_0.$$

Show that (by interchanging mixed partials) that the following equations must hold if we were to be able to find such a $\mathbf{u}$:

$$\epsilon_{nj,km} + \epsilon_{km,jn} - \epsilon_{kn,jm} - \epsilon_{mj,kn} = 0.$$