Homework 3, Due Saturday, June 14
Math 269C: Numerical Methods for Elliptic Equations

I recommend using MATLAB or Python for solving the linear systems in the programming problems, but you are welcome to use whatever platform you prefer.

1. Consider the matrix arising from a FEM discretization of linear elasticity with Neumann boundary conditions. There are three null modes for 2D problems and six for 3D problems, derive them.

2. Write a problem that discretizes the Poisson problem over the domain shown in Figure 1 below. Use \( u(x, y) = \sin(2\pi x)\cos(2\pi y) \) as your exact solution. Use Dirichlet boundary conditions over the left most wall of the domain and appropriate Neumann conditions over the remaining portion of the boundary.

3. Write a problem that discretizes the linear elasticity problem over the domain shown in Figure 1 below. Use \( \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \) and \( \mu = \frac{E}{2(1+\nu)} \) with \( E = 100 \) and \( \nu = .3 \). Plot your solution as a deformed version of the mesh. Use zero Dirichlet (displacement) boundary conditions over the left most wall of the domain and zero Neumann over the remaining portion of the boundary. Use \( f = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} \).

Figure 1: Irregular 2D domain. This can be loaded into matlab with the commands load ’mesh_with_holes.dat’ and load ’nodes.dat’. The first command creates an array (which will be a variable called mesh_with_holes after the call) with mesh connectivity information (with indexing starting from 1 not 0) and the second creates an array (called nodes) that lists the vertices in the mesh.