Homework 2
Math 269A: Numerical methods for ODEs
Due: Friday, March 2

1 Pen and paper

1. Strikwerda 3.2.1, 3.2.3.
2. Strikwerda 4.1.1.
3. Derive an error bound for standard leap frog with periodic boundary conditions

\[ \frac{u_i^{n+1} - u_i^{n-1}}{\Delta t} + a \frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x} = 0, \ i = 0, 1, \ldots, N, n = 2, 3, \ldots \]

(1)

use Lax-Wendroff to initialize the \( u_i^n \). You can assume \( \lambda = \frac{\Delta t a}{\Delta x} \) has \( |\lambda| < 1 \).


2 Programming

Consider the wave equation

\[ u_t + au_x = 0, \ x \in [0,1], \ t \in [0,T) \]

(2)

\[ u(x,0) = \begin{cases} \frac{1}{2}, & x \in [.25,.75] \\ 0, & \text{otherwise} \end{cases} \]

(3)

with \( a = .5 \) and \( T = 1.9 \).

1. Approximate the solution with forward time backward space and \( N = 100, \lambda = \frac{a \Delta t}{\Delta x} = .9 \). Plot your approximate solution at time \( T \).

2. Approximate the solution with Lax Friedrichs and \( N = 100, \lambda = \frac{a \Delta t}{\Delta x} = .9 \). Plot your approximate solution at time \( T \).

3. Approximate the solution with Lax Wendroff and \( N = 100, \lambda = \frac{a \Delta t}{\Delta x} = .9 \). Plot your approximate solution at time \( T \).