

Extra Credit Handout

- You will have to stagger the grids to get incompressible flow to work. That is basically the point of me giving you a handout here. First though, let me recap the splitting scheme. You want to solve:

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{u}}{\partial \mathbf{x}} \mathbf{u} = \frac{\mu}{\rho} \Delta \mathbf{u} + \frac{1}{\rho} \nabla p$$
$$\nabla \cdot \mathbf{u} = 0$$

Here, $\frac{\partial \mathbf{u}}{\partial \mathbf{x}}$ is 2x2 matrix (i.e. $[\frac{\partial \mathbf{u}}{\partial \mathbf{x}}]_{ij} = \frac{\partial u_i}{\partial x_j}$) and the i^{th} component of the two advection diffusion equations would have this term $[\frac{\partial \mathbf{u}}{\partial \mathbf{x}} \mathbf{u}]_i = \frac{\partial u_i}{\partial x_j} u_j$. Also, the i^{th} Laplacian term is $[\Delta \mathbf{u}]_i = \frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial y^2}$.

- The splitting gives you the following equations to solve

$$\frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} + \frac{\partial \mathbf{u}^*}{\partial \mathbf{x}} \mathbf{u}^n = \frac{\mu}{\rho} \Delta \mathbf{u}^*$$
$$\frac{\Delta t}{\rho} \Delta p^{n+1} = -\nabla \cdot \mathbf{u}^*$$
$$\mathbf{u}^{n+1} = \mathbf{u}^* + \frac{\Delta t}{\rho} \nabla p^{n+1}.$$

- Finally, this should be done on a two-dimensional periodic domain. Now, the grids need to be staggered according to the picture below:

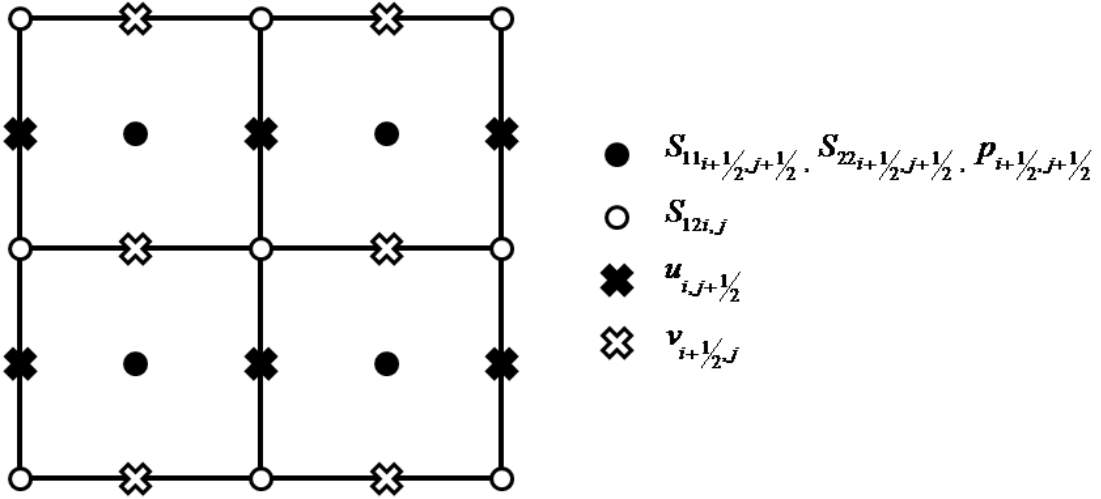


Figure 1: The unknowns here are the pressure and the two components ($\mathbf{u} = (u, v)$) of the velocity (ignore the S 's, that's for viscoelastic flow). They need to live on staggered grids (essentially so that the pressure is well defined). That is, pressure needs to be defined at cell centers, u on the vertical cell faces and v on the horizontal cell faces. Note the on the periodic domain this amounts to the same number of u , v and p unknowns. This staggering effects the convection diffusion equation though because there is coupling between the variable (just average to get them where they need them).