

M269B: HOMEWORK 5

Due Friday, Feb 27

Strickwerda: 6.1.4, 6.2.1, 6.2.3

Computational

Consider the following PDE

$$u_t = bu_{xx}, \quad x \in [0, 2], t \in (0, 1] \quad (1)$$

with initial condition

$$u(x, 0) = \sin(\pi x) + \sin(3\pi x) + \sin(7\pi x) \quad (2)$$

and periodic boundary conditions

$$u(0, t) = u(2, t), \quad t \in (0, 1]. \quad (3)$$

Use Forward-time central-space and Crank-Nicolson to solve the above numerically to final time $t = 1$ for $b = 1, 10, 100$ with $\Delta x = 1/40$. For Forward-time central-space use the largest time-step for which the scheme is stable. For Crank-Nicolson use $\Delta t = 1/80$. Compute the running time and accuracy of each run and evaluate the efficiency of each scheme. Include a graph of the initial condition and of the numerical solution at $t = .25$ and $t = 1$ with $b = 1$ for both schemes. What do you notice about the behavior of the solution?