M269B: HOMEWORK 2

Due Friday, Jan 23

Strickwerda: 2.2.1, 2.2.2, 2.2.4

Duhamel's Principle
Consider the inhomogeneous one-way wave equation

$$u_t + au_x = f(x,t), \quad u(x,0) = 0, \tag{1}$$

and assume f is such that there is a unique smooth solution $u(x,t)$ for all $t > 0$. Let $w(x,t;s)$ be the (smooth) solution of the homogenous IVP

$$w_t + aw_x = 0, \quad w(x,s;s) = f(x,s), \;\; t > s. \tag{2}$$

Show that $u(x,t) = \int_0^t w(x,t;s)ds$ solves (1).

Computational

Consider the following initial value/boundary value problem for the one-way wave equation:

$$u_t + u_x = 0, \;\; x \in (0,1], t \in (0,1] \tag{3}$$

$$u(x,0) = (1-x) \tag{4}$$

$$u(0,t) = (1-t). \tag{5}$$

Use forward-time central-space and forward-time backward-space to solve the above numerically to final time $t = 1$. Note the results of 2.2.2 on the stability of the forward-time central-space scheme. Taking the largest Δt such that the schemes are stable, report the maximum error $e_{\max} = \max_t \max_x |u_h - u_{exact}|$ of the computed solution on the spatial grids given by $\Delta x = 1/10, 1/20, 1/40, 1/80$. Using the results, what is your estimation of the order of accuracy of each method in space? Include a plot of the solution at $t = 1$ for each of the schemes at $\Delta x = 1/80$.