

Math269B: HOMEWORK 1

Due Friday, Jan 16

Strickwerda: 1.1.6, 1.2.2, 1.4.1, 1.5.3

Computational

Consider the following initial value/boundary value problem for the one-way wave equation:

$$u_t + u_x = f(x, t), \quad x \in (0, \infty), t \in (0, \infty) \quad (1)$$

$$u(x, 0) = g(x) \quad (2)$$

$$u(0, t) = h(t) \quad (3)$$

a) Use the method of characteristics to find the general solution.

b) With $f(x, t) = xe^{-t}$, $g(x) = (1 - x)$, $h(t) = (1 - t)$, use the Lax-Friedrichs scheme to solve the problem inside the domain $x \in [0, 1]$, $t \in (0, 1]$, with fixed spatial mesh width $h = 1/100$. For $\lambda = .8$ and $\lambda = 1.2$, plot the computed solution and the point-wise error, $e(x) = u_{exact}(x, 1) - u(x, 1)$, at the final time $t = 1$. Also include a plot of the exact solution at $t = 1$.