Homework 5
Math 269A: Numerical methods for ODEs
Due: Fri, December 5

1 Pen and paper

1. Problem 5.1 from Ascher and Petzold.

2 Programming

1. Write a program to visualize the Absolute stability regions of explicit Adams with $k = 2$ and $k = 3$. Do this by mapping (a discrete sampling of) the unit circle in the complex plane to the stability region.

2. Mass/spring: Consider the system

$$\frac{\partial}{\partial t} \begin{pmatrix} x(t) \\ v(t) \end{pmatrix} = \left( M^{-1} \left( -\frac{\partial e}{\partial x}(x(t)) - \gamma v(t) \right) \right), \quad t > 0$$

with $x = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix} \in \mathbb{R}^4$, $v = \begin{pmatrix} v_0 \\ v_1 \end{pmatrix} \in \mathbb{R}^4$, $x_0, x_1, v_0, v_1 \in \mathbb{R}^2$, $e(x) = \frac{k}{2} |x_1 - x_0|^2$, $M = \begin{pmatrix} m_0 & 0 \\ 0 & m_1 \end{pmatrix}$, $m_0 = \infty$, $m_1 > 0$, $k > 0$, $\gamma > 0$ and initial conditions $x_0(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $v_0(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $v_1(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(a) Let $m_1 = 1$, $k = 100$, $T = 10$ and $\gamma = \frac{1}{4} \sqrt{4km}$. Plot the Forward Euler approximation of the problem with $\Delta t = \frac{3}{2} \min \frac{-2a_i}{a_i^2 + b_i^2}$ where $\lambda_i = a_i + ib_i$ are the eigenvalues of the mass/spring matrix.

(b) Let $m_1 = 1$, $k = 100$, $T = 10$ and $\gamma = \frac{1}{4} \sqrt{4km}$. Plot the Forward Euler approximation of the problem with $\Delta t = \frac{3}{2} \min \frac{-2a_i}{a_i^2 + b_i^2}$ where $\lambda_i = a_i + ib_i$ are the eigenvalues of the mass/spring matrix.

(c) Repeat (a) and (b) with Backward Euler. Comment on your results.

3. Repeat Problem 2 with $e(x) = \frac{k}{2} \left( \ln(|x_1 - x_0|) \right)^2$, $x_0(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $v_0(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $x_1(0) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$, $v_1(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\gamma = \frac{1}{50} \sqrt{4km}$. However, rather than using $\Delta t = \min \frac{-2a_i}{a_i^2 + b_i^2}$ as the critical time step size for stability, you should suggest one.