MATH 3B MIDTERM II REVIEW

1. Taylor Approximation

Problem 1. Calculate the linear approximation to the given function f around the given point a:

(a) $f(x) = \cos(x), a = 0$

(b) $f(x) = \sin(x), a = \pi$

(c) $f(x) = \sqrt{e^x}, a = 2$

(d) $f(x) = 4x + 2, a = 10$

(e) $f(x) = 5, a = 6$

Problem 2. Find the Taylor Approximation to the given function f around the given point a to the given degree n:

(a) $f(x) = \cos(x), n = 3, a = \pi$

(b) $f(x) = 4x^3, n = 5, a = 0$

(c) $f(x) = x^{5/2}, n = 3, a = 1$

(d) $f(x) = x^2, n = 4, a = 2$

(e) $f(x) = \frac{1}{1-x}, n = 4, a = 0$

(f) $f(x) = \frac{1}{2-x}, n = 3, a = 1$

(g) $f(x) = \tan^{-1}(x), n = 3, a = 0$

(h) $f(x) = \ln(2 + 3x), n = 3, a = 2$

(i) $f(x) = e^{x^2}, n = 2, a = -1$

2. Differential Equations

Problem 3. Solve the following differential equations with the given initial conditions:

(a) $\frac{dy}{dx} = \sqrt{3x + 1}$, where $y(0) = 1$

(b) $\frac{dy}{dx} = 1 - 3N$, where $y(-1) = -2$

(c) $\frac{dy}{dx} = \frac{x+1}{y}$, where $y_0 = 2$ is $x_0 = 0$

Problem 4. Suppose that a population is modeled by the autonomous differential equation $\frac{dP}{dt} = P(20 - P)$. Find the equilibria. Are they locally stable or locally unstable? Why or why not? Then, solve this differential equation given the initial condition $P(0) = 10$.