

## MATH 32B: MIDTERM 1 REVIEW PROBLEMS

**Problem 1.** Find the average value of  $2x + y^2$  over the triangle bounded by the lines  $y = x$ ,  $y = 2 - x$ , and  $y = 0$ .

**Problem 2.** Evaluate  $\int_0^1 \int_0^1 xye^{x^2y} dy dx$ .

**Problem 3.** Find the volume of the solid cut off by the plane  $x + y + z = 1$  in the first octant.

**Problem 4.** Find the center of mass of a lamina in the first quadrant bounded by the curve  $y = 1 - x^2$  with density at any point proportional to the square of the distance from the  $y$ -axis.

**Problem 5.** Find the area enclosed by one loop of the polar curve  $r = \sin(2\theta)$ .

**Problem 6.** Find the volume of the figure bounded above by the surface  $z = \sqrt{x^2 + y^2} + 2$  above the region in the plane inside of the circle with radius 2 and center at  $(2, 0)$ .

**Problem 7.** Evaluate  $\int_0^1 \int_x^{2x} \int_0^2 2xyz dz dy dx$ .

**Problem 8.** Evaluate  $\iiint_H \sqrt{9 - x^2 - y^2} dV$ , where  $H$  is the upper hemisphere of the sphere of radius 3 centered at the origin.

**Problem 9.** Find the volume bounded by the two cylinders  $x^2 + z^2 = 1$  and  $y^2 + z^2 = 1$ .

**Problem 10.** CC Sabathia wants to get 14 hot dogs, but he only has one half-inning to do so. The average length of a half-inning is 10 minutes, and the average length of the hot dog line is 4 minutes. If it takes 1 minutes to walk to the hot dog cart (and 1 minute to walk back), what is the probability that he does not make it back in time? (You may assume that the two quantities in question can be modeled by exponential random variables)

---

### Answers

1.  $\frac{13}{6}$
2.  $\frac{e}{2} - 1$
3.  $\frac{1}{6}$
4.  $(\bar{x}, \bar{y}) = (\frac{15}{24}, \frac{2}{7})$
5.  $\frac{\pi}{8}$
6.  $16\pi$
7.  $\frac{3}{2}$
8.  $\frac{81}{2}\pi$
9.  $\frac{16}{3}$
10.  $\frac{5}{7}e^{-1/5} \approx 60\%$