MATH 32B: MIDTERM 2 REVIEW PROBLEMS

This midterm covers sections 16.9-17.4 of the textbook. There are two major topics in these sections: the change of variables for multiple integrals and line integrals.

Problem 1. Evaluate $\iint_R e^{x+y}\,dA$ where R is the region given by the inequality $|x|+|y| \leq 1$ using a proper change of variables.

Problem 2. Write down the transformations that give rise to polar, cylindrical, and spherical coordinates and find the Jacobian of each transformation.

Problem 3. Evaluate $\iint_R \sin(9x^2+4y^2)\,dA$ where R is the region bounded by the ellipse $9x^2+4y^2=1$.

Problem 4. Find the Jacobian of the transformation $x = e^{u+v}$, $y = e^{u-v}$, $z = e^{u+v+w}$.

Problem 5. Use a proper change of coordinates to find the area of the ellipse $x^2/a^2+y^2/b^2 = 1$ and the volume of the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2 = 1$.

Problem 6. Find the gradient of the functions $f(x,y,z) = e^{xy}z$ and $g(x,y) = x^2y^2 + \sin(x)\cos(y)\rangle$.

Problem 7. Sketch the vector fields $\langle x, 2y\rangle$ and $\langle 4, y+2\rangle$.

Problem 8. Evaluate the line integral $\int_C x^3 z\,ds$, where C is the curve given be $x = 2\sin(t)$, $y = t$, and $z = 2\cos(t)$ with $0 \leq t \leq \pi/2$.

Problem 9. Evaluate $\int_C x^3 y\,dx - x\,dy$ where C is the unit circle centered at the origin with clockwise orientation.

Problem 10. Evaluate $\int_C y\,dx + z\,dy + x\,dz$ where C consists of the line segments from $(0,0,0)$ to $(1,1,2)$ to $(3,1,4)$.

Problem 11. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x,y,z) = \langle e^z, xz, x+y\rangle$ and $\mathbf{r}(t) = \langle t^2, t^3, -t\rangle$ with $0 \leq t \leq 1$.

Problem 12. Is the vector field $\mathbf{F}(x,y) = \langle (1+xy)e^{xy}, e^y + x^2 e^{xy}\rangle$ conservative? If so, find a function f such that $\nabla f = \mathbf{F}$.

Problem 13. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is the line segment from $(0,2,0)$ to $(4,0,3)$ and $\mathbf{F}(x,y,z) = \langle e^y, xe^y + e^z, ye^z\rangle$, given that this vector field is conservative.

Problem 14. Use Green's Theorem to compute $\int_C \sqrt{1+x^2}\,dx + 2xy\,dy$ where C is the triangle with vertices $(0,0)$, $(1,0)$, and $(1,3)$ (with the positive orientation of course).

Problem 15. Computer $\int_C x^2 y\,dx - xy^2\,dy$ where C is the circle of radius 2 centered at the origin with counterclockwise orientation.