

## MATH 32B: FINAL EXAM REVIEW PROBLEMS

**Problem 1.** Compute  $\iint_R f(x, y) dA$ , where  $f(x, y) = \sqrt{4 - y^2} + 4$  and  $R = [-1, 2] \times [-2, 2]$  by identifying this integral of the volume of a solid.

**Problem 2.** Evaluate:

(a)  $\int_{-1}^1 \int_0^2 \frac{y}{1+x^2} dy dx$

(b)  $\iint_R 6xy(x + y^2) dA$ , where  $R$  is the rectangle with vertices  $(-2, 3), (1, 3), (-2, 6), (1, 6)$

(c)  $\iint_R xy e^{x^2 y} dA$ , where  $R = [0, 1] \times [0, 2]$ .

**Problem 3.** Evaluate:

(a)  $\int_0^1 \int_{3y}^3 e^{x^2} dx dy$

(b)  $\iint_R |x| dA$ , where  $R$  is the region where  $|x| \leq 1, |y| \leq 1$ , and  $|x| + |y| \geq 1$ .

(c)  $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} dx dy$ .

**Problem 4.** Suppose the volume of the solid in the first octant bounded by the surface  $ax + 3y + 2z = 12$  is equal to 4, where  $a > 0$ . Find  $a$ .

**Problem 5.** The area enclosed by one loop of the curve given in polar coordinates by  $r = c \cos(2\theta)$  is equal to 1, with  $c > 0$ . Find  $c$ .

**Problem 6.** Evaluate  $\int_0^\infty x^2 e^{-x^2} dx$  using the fact that  $\int_{-\infty}^\infty e^{-x^2} = \sqrt{\pi}$ .

**Problem 7.** Find the surface area of the region on the graph  $z = 11 + x^2 + \sqrt{3}y$  above that triangle with vertices  $(0, 0), (2, 0), (2, 3)$ .

**Problem 8.** Find the center of mass of the lamina in the shape of the trapezoid with  $0 \leq y \leq 2$ , between the lines  $y = 2 - x$  and  $y = 2x - 8$  such that  $\rho(x, y) = x + 2y$ .

**Problem 9.** Write a triple integral to express the volume of the region in first octant below the plane  $z = 1 - y$  and above the region in the  $xy$ -plane where  $\sqrt{x} \leq y \leq 1$ . Then calculate the value of this integral.

**Problem 10.** Find the volume of the solid below  $x^2 + y^2 + z^2 = z$  and above  $z = \sqrt{x^2 + y^2}$ .

**Problem 11.** Evaluate  $\iint_R \cos\left(\frac{y-x}{y+x}\right) dA$ , where  $R$  is the trapezoid with vertices  $(1, 0), (2, 0), (0, 2),$  and  $(0, 1)$ .

**Problem 12.** Find and sketch the gradient of the functions  $f(x, y) = (x+y)^2/4$  and  $g(x, y) = (x^2 + y^2)/4$ .

**Problem 13.** Find the work done by the force field  $\mathbf{F}(x, y, z) = \langle y + z, x + z, z + y \rangle$  on a particle moving along the line segment from  $(1, 0, 0)$  to  $(3, 4, 2)$ .

**Problem 14.** Consider the vector field  $\mathbf{F}(x, y) = \langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \rangle$ . Make sure you completely understand all the problems we have done on this vector field. This includes the homework problem on it, example 5 in the book, and the exam question.

**Problem 15.** Evaluate  $\int (2x + \cos(y^2)) dy + (y + e^{x^2}) dx$ , where  $C$  is the negatively oriented circle with center  $(0, 0)$  and radius 2.

**Problem 16.** Is  $\mathbf{F}(x, y, z) = \langle e^z, 1, xe^z \rangle$  conservative? If so, find  $f$  such that  $\mathbf{F} = \nabla f$ .

**Problem 17.** Is there a vector field  $\mathbf{F}$  such that  $\text{curl } \mathbf{F} = \langle 3x^2y^2z^4 - e^z, 1 - 2xy^3z^4, \cos(x^2) \rangle$ ? Is there a vector field  $\mathbf{G}$  such that  $\text{curl } \mathbf{G} = \langle xy^2, yz^2, zx^2 \rangle$ ?

**Problem 18.** Find parametric equations for the part of the cylinder  $y^2 + z^2 = 16$  that lies between the planes  $x = -1$  and  $x = 11$ . Find the surface area using an appropriate integral.

**Problem 19.** Verify that Stokes Theorem is true for the vector field  $\mathbf{F}(x, y, z) = \langle y^2, x, z^2 \rangle$  over the part of the paraboloid  $z = x^2 + y^2$  that lies below the plane  $z = 1$ , oriented upward.

**Problem 20.** Suppose that  $S$  is a surface and  $C$  is a curve that satisfies the hypotheses of Stokes Theorem. What is  $\int_C (f \nabla f) \cdot d\mathbf{r}$ , where  $f$  is an arbitrary function with continuous second order partials?

**Problem 21.** Find the value of  $\iint_S \mathbf{F} \cdot d\mathbf{S}$ , where  $\mathbf{F} = \langle x^3 + y \sin z, y^3 + z \sin x, 3z \rangle$ , where  $S$  is the surface of the solid bounded by the surfaces  $z = \sqrt{1 - x^2 - y^2}$ ,  $z = \sqrt{4 - x^2 - y^2}$ , and  $z = 0$ .

**Problem 22.** Let  $\mathbf{F}$  be a constant vector field, and suppose  $S$  and  $E$  are as in the Divergence Theorem. What is  $\iint_S \mathbf{F} \cdot \mathbf{n} dS$ ?

## ANSWERS

1.  $48 + 6\pi$
2. (a)  $\pi$ ; (b)  $-\frac{9963}{4}$ ; (c)  $\frac{e^2-3}{2}$
3. (a)  $\frac{e^2-1}{6}$ ; (b)  $\frac{4}{3}$ ; (c)  $\frac{e^{16}-1}{4}$
4. 48
5.  $\sqrt{\frac{8}{\pi}}$
6.  $\sqrt{\frac{\pi}{4}}$
7.  $5\sqrt{5} - 1$
8.  $(\bar{x}, \bar{y}) = (\frac{41}{14}, \frac{263}{210})$
9.  $\frac{1}{12}$
10.  $\frac{\pi}{8}$
11.  $\frac{3}{2} \sin(1)$
12.  $\nabla f = \frac{1}{2} \langle x + y, x + y \rangle$ ,  $\nabla g = \frac{1}{2} \langle x, y \rangle$
13. 24
15.  $2\pi$
16.  $f(x, y, z) = xe^z + y + C$
17. Yes, No.
18.  $96\pi$
20. 0
21.  $\frac{31\pi}{20} + \frac{7}{3}$
22. 0