

Math 32A. Quiz 7a March 14, 2006

Name:

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Write your solution on the exam sheet. Show some work and justify your answer. Do not just give the correct answer. You have 15 minutes. Good luck!

1. Let  $f(x, y) = (x + y)(1 + xy)$ . Find all the critical points of  $f(x, y)$  and determine whether each critical point is a local maximum, a local minimum, or neither.

Solution:  $f(x, y) = x + y + x^2y + xy^2$ . Hence

$f_x = 1 + 2xy + y^2$ ,  $f_y = 1 + 2xy + x^2$  and the critical points have  $x^2 = y^2$  so that  $y = \pm x$ . But when  $x = y$ , both derivatives are nonzero. Hence  $y = -x$  and then  $x^2 = y^2 = 1$ . Thus the critical points are  $(1, -1)$  and  $(-1, 1)$ .

Since  $f_{xx} = 2y$ ,  $f_{yy} = 2x$  and  $f_{xy} = 2$ , we have  $D = 4xy - 4$  and  $D = -8$  at each critical point. Hence by the second derivative test neither critical point is a local maximum or a local minimum.