

Math 32A. Quiz 2a January 24, 2006

**Solutions:**

A plane in 3-space contains the points  $(0, 0, 1)$ ,  $(1, 1, 2)$ , and  $(2, 2, 0)$ .

1. Find a non-zero vector normal (perpendicular) to this plane.

Solution: The plane contains the vectors

$$\vec{v} = \langle 1, 1, 2 \rangle - \langle 0, 0, 1 \rangle = \langle 1, 1, 1 \rangle$$

and

$$\vec{w} = \langle 2, 2, 0 \rangle - \langle 0, 0, 1 \rangle = \langle 2, 2, -1 \rangle .$$

Then a normal vector is given by  $\vec{v} \times \vec{w} = -3\vec{i} + 3\vec{j} + 0\vec{k} = \langle -3, 3, 0 \rangle$ . Note that  $\langle -1, 1, 0 \rangle$  is also a normal vector.

2. Find the equation (either vector equation or scalar equation) of the plane.

Solution: By problem 1 a normal vector is  $\langle a, b, c \rangle = \langle -1, 1, 0 \rangle$  and by hypothesis the plane contains the point  $r_0 = (0, 0, 1)$ . Thus its vector equation is

$$(\vec{r} - \vec{r}_0) \cdot \langle -1, 1, 0 \rangle = 0$$

and its scalar equation is

$$(-1)(x - 0) + 1(y - 0) + 0(z - 1) = 0$$

or more simply,  $y - x = 0$ .