Math 32A. Quiz 1a January 17, 2006

Solution:

1. Find the cosine of the angle between the diagonal of a cube and the diagonal of one of the cube's faces, when the two diagonals have the same initial point.

Solution: The angle will not change if we replace (x, y, z) by (ax, ay, az) for any $a > 0$. Hence we may assume the cube has edges of unit length. We can also assume $(0, 0, 0)$ is one vertex of the cube and we can assume the edges are parallel to the coordinate axes.

Then the diagonal is the vector $\vec{v} =< 1, 1, 1 >$ and the diagonal of one face is $\vec{w} =< 1, 1, 0 >$. These vectors have lengths $\sqrt{3}$ and $\sqrt{2}$ and dot product $\vec{v} \cdot \vec{w} = 2$. If θ is the angle, then

$$\cos\theta = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|} = \frac{2}{\sqrt{6}}.$$

The answer is the same for the diagonals of the other faces.