MATH 252A - Fall 2014 - Geometric Measure Theory

Time and Place: TBA, MWF 2:00, starting October 3.

Office hours: John Garnett MW 3:00 in MS 7941.

References:

1. Camillo De Lellis, *Rectifiable Sets, Densities, and Tangent Measures*, European Mathematical Society, Zurich Lectures in Advanced Mathematics, ISBN 978-3-03719-044-9.

2. Pertti Mattila, *Geometry of Sets and Measures in Euclidean Spaces, Fractals and rectifiability*, Cambridge studies in advanced mathematics, 44, Cambridge University Press, ISBN 0 521 46576 1 (hardback) ISBN 0 521- 65595 1 (paperback).

We say a Radon measure μ on Euclidean space $\mathbb{R}^n$ has *dimension* $\alpha > 0$ if the limit

$$\lim_{r\to 0} \frac{\mu(B(x,r))}{r^\alpha}$$

exists and is positive μ almost everywhere. Then some remarkable things happen. First, α must be an integer, $\alpha = k \leq n$. Second, there is a countable family Γ_j of $k-$dimensional Lipschitz surfaces and a Borel function f such that for every Borel set A

$$\mu(A) = \sum_j \int_{A\cap\Gamma_j} f(x)d\Lambda_k(x),$$

where Λ_k is k-dimensional Hausdorff measure.

This theorem depends on the work of many authors, Besicovitch, Marstrand, Mattila, Kirchheim, and Preiss. The course, whose only prerequisites are Mathematics 245 A and B, will cover the background needed for this theorem, the proof of the theorem, and several related results and applications.

The course will have two homework assignments.

- J. Garnett