Solutions to Homework problems 1

2.2.6 $a_n = \frac{(-1)^n}{(n+1)^2}$
The first six terms of this sequence are

$$a_0 = 1 \qquad a_1 = -\frac{1}{4} \qquad a_2 = \frac{1}{9} \qquad a_3 = -\frac{1}{16} \qquad a_4 = \frac{1}{25} \qquad a_5 = -\frac{1}{36}.$$

2.2.22 A formula for the sequence $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}$ is $a_n = \frac{n+1}{2n+3}$.

2.2.40 The first five terms of the sequence $a_n = \frac{n}{n+1}$ are

$$a_0 = 0 \qquad a_1 = \frac{1}{2} \qquad a_2 = \frac{2}{3} \qquad a_3 = \frac{3}{4} \qquad a_4 = \frac{4}{5}$$

and we see that these numbers get closer and closer to 1. In fact they get as close to 1 as we want and we thus see that the limit is

$$\lim_{n\to\infty} a_n = 1.$$

2.2.76 $\lim_{n\to\infty} \dfrac{3n^2 - 5}{n^2}$.
Since $\lim_{n\to\infty}(3n^2 - 5) = \infty$ does not exist we can not use the fourth limit law, that the limit of the quotient is the quotient of the limits. However we can simplify the fraction and calculate

$$\lim_{n\to\infty} \frac{3n^2 - 5}{n^2} = \lim_{n\to\infty} \left(\frac{3n^2}{n^2} - \frac{5}{n^2}\right) = \lim_{n\to\infty} 3 - \lim_{n\to\infty} \left(5\frac{1}{n}\frac{1}{n}\right)$$
$$= 3 - 5 \lim_{n\to\infty} \frac{1}{n} \lim_{n\to\infty} \frac{1}{n} = 3$$

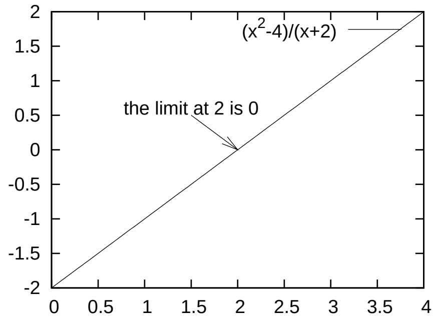
since we know that $\lim_{n\to\infty} \frac{1}{n} = 0$.

2.2.78 $\lim_{n\to\infty} \left(\dfrac{n+2}{n^2-4}\right)$. We factor the denominator and calculate

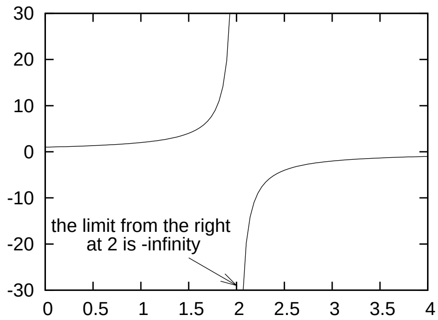
$$\lim_{n\to\infty} \left(\frac{n+2}{n^2-4}\right) = \lim_{n\to\infty} \left(\frac{n+2}{(n-2)(n+2)}\right) = \lim_{n\to\infty} \left(\frac{1}{n-2}\right) = 0.$$

The limit $\lim_{n\to\infty} \frac{1}{n-2} = 0$ is seen in the same way as seeing that $\lim_{n\to\infty} \frac{1}{n} = 0$.

3.1.14 $\lim_{x\to 2} \frac{x^2-4}{x+2}$. Note that $\frac{x^2-4}{x+2} = x-2$ everywhere except at $x=-2$. Thus



3.1.24 $\lim_{x\to 2^+} \frac{3}{2-x}$. The function $f(x) = \frac{3}{2-x}$ is discontinuous. From the graph below we see that the limit is $-\infty$.



3.1.40 $\lim_{x\to 1}(8x^3-2x+3)$ We calculate using the limit laws:

$$\begin{aligned}\lim_{x\to 1}(8x^3-2x+3) &= \lim_{x\to 1}(8x^3) - \lim_{x\to 1}(2x) + \lim_{x\to 1} 3 = 8(\lim_{x\to 1} x)^3 - 2\lim_{x\to 1} x + 3 \\ &= 8\cdot 1 - 2\cdot 1 + 3 = 9\end{aligned}$$

3.1.46 $\lim_{x\to -2} \frac{1+x}{1-x}$ We calculate using the limit laws:

$$\lim_{x\to -2} \frac{1+x}{1-x} = \frac{\lim_{x\to -2}(1+x)}{\lim_{x\to -2}(1-x)} = \frac{1+(-2)}{1-(-2)} = -\frac{1}{3}$$

3.1.48 $\lim_{x\to 2} \frac{4-u^2}{2-u}$ Since the limit of the denominator $\lim_{x\to 2}(2-u) = 0$ we cannot use the law about the quotient of limits. Instead we simplify the fraction

$$\lim_{x\to 2} \frac{4-u^2}{2-u} = \lim_{x\to 2} \frac{(2-u)(2+u)}{2-u} = \lim_{x\to 2}(2+u) = 4.$$