Math 3A Practice problems for midterm exam #1

1. Find an expression for a_n based on the values of a_0, a_1, a_2, ... for

$$\{a_n\} = 0, -\frac{2}{2}, \frac{4}{3}, -\frac{6}{4}, \frac{8}{5}, -\frac{10}{6}, \ldots$$

2. Consider the recursive sequence defined by $a_{n+1} = 2a_n - 1$ and $a_0 = 2$.

a. Write down the first five terms of this sequence.

b. Write down an explicit (non-recursive) description of this sequence.

c. Rewrite the explicit description so that the domain of the sequence is $n = 1, 2, 3, \ldots$.

3. Compute

$$\lim_{n\to\infty} \frac{n}{3n+1}$$

4. Compute

$$\lim_{x\to 1} \frac{1 - 2x + x^2}{1 - x}$$

5. Use the limit laws to compute

$$\lim_{x\to 0} \frac{x + \sin(x)}{2x}$$

6. Let $f(x)$ be defined by

$$f(x) = \begin{cases} \frac{x^2+2x-8}{x-2} & x \neq 2 \\ a & x = 2 \end{cases}$$

What value of a will make $f(x)$ continuous at $x = 2$?

7. For what x values is the function $f(x) = \sin^2(x)$ continuous?

8. Use the intermediate value theorem to show that $f(x) = x^5 + 3x^2 + 1 = 0$ has a root.

9. Graph the function $f(x) = |x^2 - 3|$, and use the graph to guess where the function is not differentiable.

10. Let a be a constant, and let $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist. Use the limit laws to show that

$$\lim_{x\to c}(f(x) - a \cdot g(x)) = \lim_{x\to c}(f(x)) - a \cdot \lim_{x\to c}(g(x))$$

11. Use the definition of the derivative to find the derivative of $f(x) = \frac{1}{x}$.

12. Given that for $e \leq x \leq e^{\sqrt{x}}$ whenever $x > e$, use the sandwich theorem to show that

$$\lim_{x \to \infty} \frac{\ln(x)}{x} = 0$$

13. The limit

$$\lim_{h \to 0} \frac{\frac{1}{(2+h)^2+1} - \frac{1}{5}}{h}$$

represents the derivative of a function at the point $(a, f(a))$. Find the function $f(x)$ and the point a.