

Math 151A Homework #6 – due Wednesday 12/06, in class

Show all your work!

1. Composite midpoint rule

Derive the expression for the composite midpoint rule (theorem 4.6). The derivation is similar to the derivation of the composite Simpson's rule, which we did in class.

2. Gaussian quadrature

According to theorem 4.7, we can approximate the integral of a function as

$$\int_{-1}^1 f(x)dx \approx \sum_{i=1}^n c_i f(x_i)$$

where

$$c_i = \int_{-1}^1 \prod_{\substack{j=1 \\ j \neq i}}^n \frac{x - x_j}{x_i - x_j} dx$$

and $\{x_j\}$ are the roots of the n th Legendre polynomial.

Show that for $n = 1$ this gives the midpoint rule

$$\int_a^b f(x)dx = (b - a)f\left(\frac{b + a}{2}\right)$$

3. Gaussian elimination [Computational]

Implement the Gaussian elimination with partial pivoting algorithm (algorithm 6.2) and use it to solve the following system

$$\begin{aligned} 3.03x_1 - 12.1x_2 + 14x_3 &= -119 \\ -3.03x_1 + 12.1x_2 - 7x_3 &= 120 \\ 6.11x_1 - 14.2x_2 + 21x_3 &= -139 \end{aligned}$$