Math 151A Homework #5 – due Wednesday 11/29, in class

Show all your work!

1. Forward and backward differences
Suppose we use Newton's divided differences on the points $x_0 = 0$, $x_1 = 1$ and $x_2 = 2$ to construct the following matrix

$$\begin{bmatrix} & 0 & 0. \\ & & 0 \\ 6 & 2 & \frac{1}{2} \end{bmatrix}$$

a. Fill in the missing elements.

b. Use the Newton *foward* difference formula to estimate $f(0.1)$.

c. Use the Newton *backward* difference formula to estimate $f(1.9)$.

2. Numerical differentiation
Let $f(x) = e^x$. Approximate the value of $f'(1)$ using:

a. The 3-point forward difference formula with $h = 0.1$

b. The 3-point backward difference formula with $h = 0.1$

c. The 3-point centered difference formula with $h = 0.1$

Which do you expect to be a better approximation, and why?

3. Richardson's extrapolation (problem 9 of section 4.2)
Suppose that $N(h)$ is an approximation to M at every $h > 0$ and that

$$M = N(h) + K_1 h + K_2 h^2 + K_3 h^3 + \dots,$$

for some constants K_1, K_2, K_3, ... Use the values $N(h)$, $N(h/3)$, and $N(h/9)$ to produce an $\mathcal{O}(h^3)$ approximation to M.

4. Numerical integration (problems 1a, 3a, 5a, 7a, 9a, and 11a of section 4.3)
Approximate the integral

$$\int_{0.5}^{1} x^4 dx$$

using

a. The trapezoidal rule

b. Simpson's rule

c. The midpoint rule

For each of the above, find a bound for the error using the error formula, and compare this to the actual error.