Math 151A Homework #4 – due Wednesday 11/01, in class

Show all your work!

1. Error term for Taylor's theorem (section 3.1, problem 31)
In class we proved Theorem 3.3, which stated the error term for the Lagrange interpolating polynomial. Using the same procedure, prove Taylor's theorem (Theorem 1.14). You will want to use the function

$$g(t) = f(t) - P(t) - [f(x) - P(x)] \cdot \frac{(t - x_0)^{n+1}}{(x - x_0)^{n+1}}$$

where $P(x)$ is the nth Taylor polynomial.

2. Lagrange interpolation (section 3.1, problems 1a & 3)
Given the function $f(x) = \cos x$, use the points $x_0 = 0$, $x_1 = 0.6$ and $x_2 = 0.9$ to do the following:

a. Construct Lagrange polynomials of degree at most one and at most two to approximate $f(x)$.

b. Use these polynomial approximations to compute the value of $\cos(0.45)$ and the absolute error.

c. Use theorem 3.3 to calculate an error bound for these approximations. Do the answers from part (b) obey the error bounds?

3. Neville's method [computational] (section 3.1, problem 11)
Use Neville's method to approximate $\sqrt{3}$ with the following functions and values

a. $f(x) = 3^x$ and the values $x_0 = -2$, $x_1 = -1$, $x_2 = 0$, $x_3 = 1$ and $x_4 = 2$.

b. $f(x) = \sqrt{x}$ and the values $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 4$ and $x_4 = 5$

c. Compare the accuracy of the approximation in parts (a) and (b).

4. Newton's divided difference [computational] (section 3.2, problem 7a)
Use Algorithm 3.2 to construct the interpolating polynomial of degree three for the unequally spaced points given in the following table:

x	f(x)
-0.1	5.300
0.0	2.000
0.2	3.190
0.3	1.000