Math 151A Homework #3 – due Wednesday 10/25, in class

Show all your work!

1. Trinary computing.
Imagine we've invented a trinary (base 3) computer, which stores information as *trits* (**tri**nary b**its**) which can take on values $0, 1$ or 2. If (as with binary computers) the trinary computer uses 52 trits to hold the mantissa (a.k.a. the trinary fraction), how many decimal digits of precision can we expect?

2. k-digit rounding approximation (Problem 24 of section 1.2)
Suppose that $fl(y)$ is a k-digit rounding approximation to y. Show that

$$\left|\frac{y - fl(y)}{y}\right| \le 0.5 \times 10^{-k+1}.$$

[*Hint:* If $d_{k+1} < 5$, then $fl(y) = 0.d_1 d_2 \dots d_k \times 10^n$. If $d_{k+1} \ge 5$, then $fl(y) = 0.d_1 d_2 \dots d_k \times 10^n + 10^{n-k}$.]

3. Approximating π
Compute the absolute error and relative error for approximating π by $22/7$. How many significant digits is the approximation good to?

4. Approximating e

a. The number e can be approximated by the sequence

$$e \approx \sum_{k=0}^{n} \frac{1}{k!}.$$

If $n = 4$, what is the relative error and number of significant digits in this approximation of e? How about when $n = 8$?

b. The number e can also be approximated by

$$e \approx n \cdot \left(\frac{\sqrt{2\pi n}}{n!}\right)^{1/n}$$

What is the relative error and number of significant digits when $n = 4$, $n = 8$, and $n = 20$?