

Math 151A Homework #2 – due Wednesday 10/18, in class

Show all your work!

1. The square root of two, revisited.

Use Newton's method and the secant method to approximate the value of $\sqrt{2}$ to within 10^{-4} . As usual, submit your code and a list of intermediate approximations.

- Use Newton's method with an initial guess $p_0 = 1$. How does this compare with the bisection method (from last week's homework), in terms of number of iterations required?
- Use Newton's method with an initial guess $p_0 = 0.0001$. What happens to the number of iterations required, and why?
- Use the secant method with initial guesses $p_0 = 1$ and $p_1 = 2$. How does this compare to the Newton's method from part a?

2. Linear convergence. (problem 6b of section 2.4)

Show that the sequence defined by $p_n = 1/n^2$ (for $n \geq 1$) converges linearly to $p = 0$. How large must n be before we have $|p_n - p| \leq 5 \times 10^{-2}$?

3. Quadratic convergence. (problem 8, section 2.4)

Show that the sequence defined by $p_n = 10^{-2^n}$ converges quadratically to 0.

4. Dealing with multiple zeros.

Consider the function $f(x) = x^3 + 4x^2 + 8x$.

- Show that $p = 0$ is a simple zero of $f(x)$.
- Use Newton's method to find the root to within 10^{-6} , with initial guess $p_0 = 1$.

Now consider the similar function $g(x) = x^3 + 4x^2$.

- Show that $p = 0$ is a zero of multiplicity 2 of $g(x)$.
- Use Newton's method to find the root to within 10^{-6} , with initial guess $p_0 = 1$.
- Use the modified (multiple zero) Newton's method to find the root to within 10^{-6} , with initial guess $p_0 = 1$.