

Math 151A Homework #1 – due Wednesday 10/11, in class

Show all your work!

1. On the importance of stopping criteria. (section 2.1, problem 16)

Let $f(x) = (x - 1)^{10}$ (with a root at $p = 1$), and consider the sequence of approximations to the root p given by $p_n = 1 + (1/n)$. Show the the stopping criterion $|f(p_n)| < 10^{-3}$ is met for $n \geq 2$, but $|p - p_n| < 10^{-3}$ requires that $n > 1000$.

2. On the importance of stopping criteria, II. (section 2.1, problem 17)

Let p_n be the sequence defined by $p_n = \sum_{k=1}^n (1/k)$. Show that p_n diverges even though $\lim_{n \rightarrow \infty} (p_n - p_{n-1}) = 0$.

3. The square root of two.

Use the bisection method to approximate the value of $\sqrt{2}$, given that it lies in the interval $[1, 2]$. Do this in two parts:

- a. Calculate the maximum number of iterations required to estimate $\sqrt{2}$ to within 10^{-4} , starting with the interval $[1, 2]$.
- b. Use the bisection method to compute the value of $\sqrt{2}$ to within 10^{-4} . Print out and submit (along with your code) the intermediate approximations at each iteration.

[Hint: Consider $f(x) = x^2 - 2$]