

Math 142 Practice problems for midterm exam

1. Phase plane sketch Consider a system with the potential $V(x) = x^2 - x^4/4$.

- Find the stable and unstable equilibrium points.
- Sketch the trajectories in the phase plane.
- Consider the potential $V(x) = x^2 - x^4/4 + 2$. Why does this potential produce the same dynamics?

2. Damped harmonic oscillator If you were designing a suspension system for a car, would you want the spring system to be underdamped, overdamped, or critically damped and why?

3. Nonlinear pendulum Consider the non-linear pendulum $\ddot{\theta} = -\frac{g}{\ell} \sin \theta$.

- Compute the potential energy, using the above equation.
- The potential energy of a mass under the influence of gravity is given by $V(y) = mg(y - y_0)$. Use this to compute then potential energy of the non-linear pendulum.
- Are they the same?
- Redo parts *a* and *b* for the non-dimensionalized non-linear pendulum.

4. Exponential growth We saw in class two types of exponential growth: continuous and discrete. Suppose a population grows discretely with a yearly growth rate of R . If we modelled this with a continuous growth process, what instantaneous growth rate would produce the same doubling time?

5. Newton's law If a mass m is launched with initial velocity v_0 at an angle θ with respect to horizontal, how far does it go? What angle would produce the maximum distance?