

## **Mathematics 33A, Practice Midterm, October 12, 2009.**

Calculators, books, or notes of any kind are not allowed on the exam.

There are 11 items on this practice exam altogether, and they all have equal value. Answer as many of them as you can in **50 minutes**. You must show your work in all questions.

The questions are not always arranged in order of difficulty. Look through them when you start so you get an idea of the time you'll need. If you're not sure what to do on an item then move onward and return to it later. (Some of the questions can be solved with geometric reasoning instead of lengthy computations; this may save you some time.)

After taking the practice test, grade yourself as follows: For each fully correct answer give yourself 5 points. For a score at the A range you should aim to have fully correct answers for at least 9 questions, within the allotted 50 minutes.

**Good luck.**

**Question 1.** Solve the system of equations  $3x + 2y + z = 5$ ;  $x + y - z = 3$ .

**Question 2.** Find the inverse of  $\begin{pmatrix} 1 & 5 & 3 \\ 2 & 11 & 6 \\ 1 & 5 & 4 \end{pmatrix}$ .

**Question 3.** Let  $A$  be the matrix of a rotation  $25^\circ$  counterclockwise, and let  $B$  be the matrix of a rotation  $35^\circ$  counterclockwise (both around the origin, in  $\mathbb{R}^2$ ). Describe  $BA$  geometrically. Use this to find the matrix  $BA$ .

**Question 4.** Find each of the following products, if it makes sense. If it doesn't then write "does not make sense."

(a)  $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$

(b)  $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

**Question 5.** Find the kernel of the orthogonal projection to the line  $x_2 = 2x_1$  in  $\mathbb{R}^2$ . (You can reason geometrically.)

**Question 6.** Find vectors which span the kernel of the linear transformation  $T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = x_1 + x_2 + x_3$  from  $\mathbb{R}^3$  to  $\mathbb{R}$ .

**Question 7.** Give an example of a linear transformation whose image is the plane  $x_1 + x_2 + x_3 = 0$  in  $\mathbb{R}^3$ .

**Question 8.** For each of the linear transformations below, determine whether it is invertible. If it is then describe its inverse (geometrically). If it is not invertible then explain why.

(a) Rotation  $30^\circ$  counterclockwise around the origin in  $\mathbb{R}^2$ .

(b) Reflection along the line  $x_2 = 3x_1$  in  $\mathbb{R}^2$ .

(c) Orthogonal projection to the line  $x_2 = 3x_1$  in  $\mathbb{R}^2$ .