Mathematics 33A, First Midterm, October 16, 2009.

Calculators, books, or notes of any kind are not allowed on this exam. Do not use any paper other than that provided. (You may write on the back if you need more space, but indicate this clearly on the front.)

There are 11 items on this exam altogether, and they all have equal value. Answer as many of them as you can. The questions are not always arranged in order of difficulty. Look through them when you start so you get an idea of the time you'll need. If you're not sure what to do on an item then move onward and return to it later. (Some of the questions can be solved with geometric reasoning instead of lengthy computations; this may save you some time.) **Good luck.**

Family Name: ______________________

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Discussion section: ______________________

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Question 1. Solve the system of equations $\left\{ \begin{array}{rcl} x + 2y - z + 2w & = & 3 \\ 3x + 6y - z & = & 5 \end{array} \right.$.

Question 2. Compute the product $\begin{pmatrix} -1 & 0 \\ 0 & 1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}$.

Question 3. Let $A = \begin{pmatrix} \cos 60^\circ & -\sin 60^\circ \\ \sin 60^\circ & \cos 60^\circ \end{pmatrix}$. Find all matrices $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ so that $B \cdot A = A \cdot B$. Interpret your answer geometrically.

Question 4. Find the inverse of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Question 5. Let A be the matrix of orthogonal projection to the line $y = 3x$ in $\mathbb{R}^2$, and let B be the matrix of orthogonal projection to the line $y = -\frac{1}{3}x$. (The two lines are perpendicular.) Find $B \cdot A$.

Question 6. Let A be the matrix of reflection about the line $y = 3x$ in $\mathbb{R}^2$, and let B be the matrix of reflection about the line $y = -\frac{1}{3}x$. Find $B \cdot A$.

Question 7. For each of the following, give an example if there is one, and otherwise write "none".

(a) A 2×2 matrix A so that kernel(A) = image(A).

(b) A linear transformation T whose kernel is the span of the vector $\begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$.

Question 8 Use Gauss-Jordan elimination (row operations) to determine the number of solutions of the following system. For which k is there a unique solution? infinitely many? none?

$$\begin{cases} x + y + z = 1 \\ x + 2y + kz = 2 \\ x + 4y + k^2 z = 3 \end{cases}$$

Question 9. Find a matrix A so that $A \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, $A \cdot \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, and $A \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$.

Question 10. Find the matrix of orthogonal projection to the line $y = 5x$ in $\mathbb{R}^2$.