Math 33B, Lecture 2 Midterm II Solutions

1. Set $y = C_1 \sin 2x + C_2 \cos 2x$ and plug into the DE to obtain $C_1 = -1/3$ and $C_2 = -\sqrt{3}/3$. Thus

$$y_p = -1/3 \sin 2x - \frac{\sqrt{3}}{3} \cos 2x = -2/3 \sin(2x + \pi/3) = 2/3 \sin(2x + 4\pi/3).$$

• To obtain θ: for example with $A = -2/3$, we look for $\theta \in [0, 2\pi]$ such that $\sin\theta = \frac{\sqrt{3}}{2}$ and $\cos\theta = \frac{1}{2}$. This gives you $\theta = \pi/3$. (Note that solving for $\tan\theta = \sqrt{3}$ give you two values of $\theta = \pi/3, 4\pi/3$ and thus this is not enough to decide on θ.)

2. Two homogeneous solutions are $y_1 = x$ and $y_2 = 1/x^2$. Set $y = y_1 u_1 + y_2 u_2$ to obtain

$$x u_1' + x^{-2} u_2' = 0, \quad u_1' + 2x^{-3} u_2' = e^x.$$

It follows that $u_2' = x^3 e^x$ and $u_1' = -e^x$.

Hence one obtains

$$y = C_1 x + C_2 x^{-2} - x e^x + x^{-2} \int x^3 e^x dx.$$

3.

let $y = e^{rx}$ and plug into equation, one gets $(r+1)^2 = \lambda$. Hence $r = -1 \pm \sqrt{-\lambda}$ if $\lambda \leq 0$ or $r = -1 \pm \sqrt{\lambda} i$ if $\lambda > 0$. One should check that if $\lambda \leq 0$ then the only solution to the BVP given is $y = 0$(hence no eigenvalues/eigenfunctions). When $\lambda > 0$, the eigenvalues(λ for which a nonzero solution exists) are $\lambda_n = (n\pi)^2$ with associated eigenfunctions $e^{-x} \sin n\pi x$.

4. (a) $x = 0$ is regular singular since $xP(x) = 2/3$ and $Q(x) = 1/3x$ are both analytic at $x = 0$.

(b) indicial equation:

$$r(r-1) + 2/3r = r(r-1/3) = 0.$$

(c) There are two lin. independent series solutions about $x = 0$ since the two roots of indicial equation differs by noninteger.