Math 33B, Lecture 2 Midterm I Solutions

1. Integration factor: $\sqrt{t^2-3}$. Equation becomes

$$\frac{d}{dt}(\sqrt{t^2-3}y) = \frac{3t}{\sqrt{t^2-3}}$$

Thus the general solution is

$$y(t) = (t^2-3) + \frac{C}{\sqrt{t^2-3}}$$

Plug in $y = 1, t = 4$ and one gets $C = -12\sqrt{13}$.

2. (25 pts) (a) Since ODE above must be exact the following equation holds.

$$\frac{\partial M}{\partial x} = \frac{\partial (x^4 - ysinx)}{\partial y} \tag{1}$$

So, $\frac{\partial M}{\partial x} = -sinx$, after integrating we have $M(x) = cosx + c$, where c is a constant. Using initial condition $1 = M(0) = cos0 + c \Longrightarrow c = 0$. Hence

$$M(x) = cosx \tag{2}$$

(b) Let us denote by $f(x, y)$ the solution of the ODE, then

$$\frac{\partial f}{\partial y} = cosx \tag{3}$$

and

$$x^4 - ysinx = \frac{\partial f}{\partial x} = -ysinx + g'(x) \tag{4}$$

¿From equation (1.3) we have that $f(x, y) = \int cosdy + g(x) = ycosx + g(x)$. And equation (1.4) implies that $g'(x) = x^4 - ysinx + ysinx = x^4 \Longrightarrow g(x) = \frac{x^5}{5}$. Hence,

$$f(x, y) = ycosx + \frac{x^5}{5} \Longrightarrow \tag{5}$$

$$c = ycosx + \frac{x^5}{5} \tag{6}$$

3.

(a) $k = 1$ since outgoing amount of mixed water is $(k-2)t$ and $120 + (k-2) \times 120 = 0$.

(b)

$$\frac{dA(t)}{dt} = 2 - \frac{2A(t)}{120 - t}.$$

4.

(a) The solution has to be a linear combination of $\sin(2n\pi t)$ and $\cos(2n\pi t)$, where n is an integer. Therefore $b = 0$ and $k = -4n^2\pi^2$, where n can be any integer.

(b) $y = C_1 e^{7t} + C_2 e^{-t}$.

(c) Since $y \to \infty$ as $t \to \infty$ and $t \to -\infty$, y has to be a form of $y = C_1 e^{m_1 t} + C_2 e^{m_2 t}$ where $C_1, C_2 > 0$ and m_1 and m_2 are of opposite sign. This is the case if and only if $k > 0$.