

### Additional Review problems for Midterm II

• Please explain all of your answers. We will only give partial credit to the answers given without explanation of the reasoning.

1. Consider the following differential equation

$$y'' + by' + ky = \sin 2x + \cos 2x$$

where  $b \geq 0$ .

(a) Suppose that one solution oscillates and the size of the solution grows as  $x$  grows. Find  $b$  and  $k$ .

(b) Solve for  $b = 0$ ,  $k = -1$  and find the phase angle and maximum of the particular solution obtained by undetermined coefficients method.

2. Solve  $x^2y'' + xy' - y = \tan x$  by variation of parameters. Leave the integral terms if necessary. (Hint: we know how to obtain homogeneous solutions from 4.7)

3. Find the recurrence relation for series solutions of  $(x - 1)^2y'' + xy = 0$

(a) about  $x = 0$ , (b) about  $x = 1$ .

4. Consider

$$x^2y'' + 2 \sin xy' - 3e^x y = 0.$$

We look for solutions of the form

$$(*) \quad y = \sum_{n=0}^{\infty} c_n x^{n+r}.$$

(a) Is  $x = 0$  regular singular point? why?

(b) Find the indicial equation.

(c) Is it guaranteed that there is two linearly independent solutions of the form (\*)? Explain.

**Answers to the problems 1.** (a) The only possible such solution is in the form of  $C_1x \sin 2x + C_2x \cos 2x$  (resonance). This happens when  $b = 0, k = 4$ .

(b) particular solution

$$y = -1/5 \sin 2x - 1/5 \cos 2x = -\frac{\sqrt{2}}{5} \sin(2x + \pi/4).$$

Phase angle:  $\pi/4$ . Maximum  $\frac{\sqrt{2}}{5}$  obtained at  $x = n\pi + \pi/8$ .

2. The homogeneous solutions are  $y_1 = 1/x$  and  $y_2 = x$ . Then we use the variation of parameter (see chapter 4.6) to solve the equation: we obtain

$$y = \frac{1}{2x} \int \tan x dx + \frac{x}{2} \int \frac{\tan x}{x^2} dx.$$

3. Note that  $x = 1$  is regular singular point (apply 6.2) and  $x = 0$  is ordinary point (apply 6.1). Answer is omitted.

4. (a) Yes.  $xP(x) = \frac{2 \sin x}{x}$  and  $x^2Q(x) = -3e^x$  are analytic at  $x = 0$ .

(b)  $r(r - 1) + 2r - 3 = 0$  (or  $r^2 + r - 3 = 0$ .)

(c) Yes. The two roots  $r_1, r_2$  of the indicial equation are different and do not differ by an integer.