285G: Seminar in Analysis

Time and Place: MWF 2-2:50pm, Online-recorded

Topic: Transport with density constraint

We will study various transport equations where the density is transported under a height constraint. The motivation comes from crowd motion in congested environment, and also from various tumor growth models. The problem usually features a congested region, where the density is saturated at maximum value, which evolves over time with Darcy's law. This rather simple problem leads to many interesting phenomena and provides link to several classical fluid interface problems, such as Hele-Shaw and Muskat problem. We will discuss some of these problems which have been very actively studied in the past decade in different communities. In particular we will discuss:

(a) Crowd motion models, and derivation of the interface problem based on the optimal transport approach.

(b) Tumor growth models, and derivation of the interface problem based on weak solutions approach.

(c) Multi-density problems and challenges associated with them.

(d) Associated literature on fluids models.

The class will cover a brief self-contained introduction of some central tools in the literature, which includes the theory of viscosity solutions and minimizing movements.

In the later half of the course the participants would be assigned on presentations of articles depending on their preference.

References (tentative) will include:

- Bertrand Maury, Aude Roudneff-Chupin, and Filippo Santambrogio, *A macroscopic crowd motion model of gradient flow type.* Mathematical Models and Methods in Applied Sciences, 20(10):1787–1821, 2010.

- Benoit Perthame, Fernando Quiros, and Juan Luis Vazquez, *The Hele-Shaw asymptotics for mechanical models of tumor growth.* Arch. Ration. Mech. Anal., 212(1):93–127, 2014.

- D Alexander, I Kim and Y Yao, *Quasi-static evolution and congested crowd transport,* Nonlinearity, 27 (4), p823.

- N. Guillen, I. Kim and A. Mellet, *A Hele-Shaw limit without monotonicity*, arXiv:2012.02365.

- F. Otto, *Evolution of Microstructure in Unstable Porous Media Flow: A Relaxational Approach,* Comm. Pure Appl. Math, 52 (1999), 873 – 915.

- A. Kiselev and Y. Yao, *Small scale formations in the incompressible porous media equation,* https://arxiv.org/abs/2102.05213