Math 266B Winter 2016: Homework 8. Due 3/11 in class.

1-3. Evans p. 165. Problem 17,19,20.

4. [Finite propagation property] Let u be a nonnegative solution of the porous medium equation

$$(PME_m) \qquad u_t - \Delta(u^m), \quad m > 1$$

such that u is smooth in $\{u > 0\}$.

(a) Let us assume that $v = \frac{m}{m-1} u^{m-1}$ is differentiable in $\overline{\{v > 0\}} = \overline{\{u > 0\}}$. Verify that for u as the Barenblatt solutions (given in (35)-(36) in p186 Evans) the assumption holds.

(b) Show that, for any point $(x, t) \in \partial\{u > 0\}$ with $t > 0$, the outward normal velocity of the set $\{u(\cdot, t) > 0\}$ at x is given by $|Dv|(x, t)$ (Hint: compute the PDE that v satisfies)