Math 266B Winter 2016: Homework 3. Due 1/29.

1. Let U be a bounded domain in $\mathbb{R}^n$ with smooth (C^2) boundary and let $g \in C(\partial U)$. $u(x)$ be as defined in Perron's Method, that is

$$u(x) := \sup\{v(x) : v \text{ is subharmonic } \textit{and} - M \leq v \leq g \text{ on } \partial U\}.$$

for a sufficiently large $M > 0$. In class we have shown that u is harmonic in U.

(a) Let us fix a boundary point $x_0 \in \partial U$. Construct an explicit function $h \in C(\bar{U})$ such that h is harmonic in U, $h(x_0) = 0$ and $h < 0$ on $\partial U - \{x_0\}$ (Hint: use exterior ball property of U at x_0).

(b) Using (a), show that u is continuous up to the boundary, that is, $u(x)$ converges to $g(x_0)$ as $x \in U$ converges to $x_0 \in \partial U$.

2-4. Evans p. 85. Problem 14,15 and 16.

5. Let $U = B(0, R)$ be a ball of radius R in $\mathbb{R}^n$ with center at origin, and let u be a nonnegative smooth solution of

$$u_t - \Delta u = f(x, t) \text{ in } U_T,$$

where f satisfies $\max_{U_T} f(x, t) \leq M$.

(a) Let $x = (x_1, ...x_n)$. Show that $w(x, t) = u(x, t) - e^{R+x_1} M$ satisfies $w_t - \Delta w \leq 0$ in U_T.

(b) Conclude that

$$\sup_{U_T} u \leq \sup_{\Gamma_T} u + (e^{2R} - 1)M.$$