

**Math 266B: Homework 7, due 03/04**

1-4. Evans p. 163, problem 10, 13 (please assume  $H$  is  $C^2$  and strictly convex, and  $g$  is Lipschitz), 14, 16.

5. (vanishing viscosity method) Here we discuss a popular method of regularizing first order equations by adding small diffusion. Consider the equation  $u_t - \varepsilon u_{xx} + u_x^2 = 0$  with initial data  $u = g$ , where  $g$  is smooth.

- (a) Show that  $\omega = \exp^{-u/\varepsilon}$  satisfies the heat equation  $\omega_t - \varepsilon \Delta \omega = 0$  in  $\mathbb{R} \times (0, \infty)$ .
- (b) Using (a), derive an explicit formula for  $u$ . Note that  $u$  is smooth for any  $\varepsilon > 0$ .
- (c) Now let  $\varepsilon \rightarrow 0$ : can we show that the solution converges?