Math 266B: Homework 5. Due Feb.12th.

1. Evans p88, problem 19. (d).

2. Evans page 90, problem 23.

3. Consider a positive definite, symmetric $n \times n$ matrix A, and consider the anisotropic wave equation

$$u_{tt} - \nabla \cdot (A \nabla u) = 0 \text{ in } \mathbb{R}^n \times (0, \infty).$$

Using the energy $E(t) = \int_{|x| \leq r(t)} [(u_t)^2 + \nabla u \cdot A \nabla u](x,t) dx$ over appropriate function $r(t)$, find the domain of dependence, i.e, state and prove a corresponding theorem to Theorem 6 , page 84 of Evans. The answer should be given in terms of certain eigenvalue of A.

4. Let $u(x,t)$ solve the wave equation $u_{tt} - \Delta u = 0$ in the domain

$$D = \{(x,t) \in \mathbb{R}^n \times \mathbb{R} : |x_n| < t\}.$$

Suppose that $u = 0$ on the boundary surfaces $x_n = t$ and $x_n = -t$, and $u = 0$ when $|x| > R$ for sufficiently large R. Show that u is identically zero in D.