Math 266B Winter 2016: Homework 1. Due 1/15

1. Let Ω be an open and bounded set in $I\!R^n$ and let $f : \Omega \to I\!R$. We assume that f is continuous and satisfies

$$\int_\Omega f(x)g(x)dx = 0$$

for any g compactly supported in Ω. Show that then f is identically zero in Ω.

2. Suppose $f(x,t)$ and $g(x)$ are smooth in their respective domains. Write down an explicit formula for the function u solving the initial value problem

$$\begin{cases} u_t + b \cdot Du + cu = f & \text{in} \quad I\!R^n \times (0,\infty) \\ u = g & \text{on} \quad I\!R^n \times \{t=0\}. \end{cases}$$

3. Evans p85, Problems 3.

4. An integrable function in a domain U is called *weakly harmonic* in U if

$$\int_U u\Delta\phi dx = 0$$

for all functions $\phi \geq 0$ in $C^2(U)$ with compact support in U. Show that a continuous weakly harmonic function in U is harmonic in U. (Hint: use mollifiers)

5-6. Evans p85, Problems 4 -5.