Math 266B Winter 2013: Homework 5. Due 2/13 in class.

1-3. Evans p. 88. Problem 19 (d), 23, 24.

4. Let U be an open, bounded domain in $\mathbb{R}^n$ and let $u(x,t)$ solve the wave equation in an inhomogeneous elastic medium:

$$u_{tt} - \Delta u - q(x)u = 0 \text{ in } U \times (0,T].$$

Here $q(x) \geq 0$ represents the elasticity of the medium at x. Show that solutions in $C^2(U \times [0,T])$ are uniquely determined by their data on Γ_T.

5. Use Duhamel's principle to find the solution of the nonhomogenous wave equation $u_{tt} - \Delta u = f(x,t)$ in $\mathbb{R}^3 \times [0,\infty)$ with initial conditions $u(x,0) = u_t(x,0) = 0$. What regularity in $f(x,t)$ is required for the solution u to be C^2?

6. Let $u(x,t)$ solve the hyperbolic equation

$$u_{tt} - \nabla \cdot (A\nabla u) = 0,$$

where $A = A(x)$ is a real, symmetric, positive definite matrix, with its biggest eigenvalue λ.

(a) Show that for any two vectors $v, w \in \mathbb{R}^n$, $2|Av \cdot w| \leq \lambda^{1/2}(|w|^2 + v \cdot Av)$.

(b) Using (a), state a theorem corresponding to Theorem 6, replacing the cone C with a different one depending on λ.