

Math 266B: Homework 7, due 03/06

1-4. Evans p. 163, problem 10, 13 (please assume H is C^2 and strictly convex, and g is Lipschitz), 14, 16.

5. (vanishing viscosity method) Here we discuss a popular method of regularizing first order equations by adding small diffusion. Consider the equation $u_t - \varepsilon u_{xx} + u_x^2 = 0$ with initial data $u = g$, where g is smooth.

- (a) Show that $\omega = \exp^{-u/\varepsilon}$ satisfies the heat equation $\omega_t - \varepsilon \Delta \omega = 0$ in $\mathbb{R} \times (0, \infty)$.
- (b) Using (a), derive an explicit formula for u . Note that u is smooth for any $\varepsilon > 0$.
- (c) Now let $\varepsilon \rightarrow 0$: can we show that the solution converges?